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AND

DISCUSSIONS

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NOTE.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

PIPE-LINE FLOW OF SOLIDS IN SUSPENSION

A SYMPOSIUM

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **February, 1942.** Opening discussion on this Symposium appears elsewhere in this issue.

EFFECT ON THE CHARACTERISTICS OF CENTRIFUGAL PUMPS

By Leigh C. Fairbank, Jr.,[1] Esq.

Synopsis

The centrifugal pump, by merit of its principle and construction, offers one of the best means for pumping liquids containing relatively high concentrations of suspended material, as is evidenced by its wide use in dredging and mining operations and various industrial processes. However, the assumption has often been made that the total head developed by a centrifugal pump at a given discharge is independent of the concentration of foreign material carried by the liquid being pumped. Tests have proved that this assumption is erroneous, but no methods have been developed to predict this change in head. The design and choice of a pump to meet the conditions of any particular job are based upon the past performance of other pumps, operating under similar conditions, and quite frequently on the clear-water characteristics of a standard pump; this latter method, however, amounts to nothing more than a very rough estimate.

The effect of suspended material upon the head-capacity characteristics is of particular importance in the dredging field since the performance of a pipe-line dredge is dependent on the characteristics of the various units of the dredge, the prime mover, the pump, the pipe line, and the cutter head. Since the type of material handled and the head developed may vary over a wide range, it is necessary to know the effect of various materials upon the individual units if an accurate prediction or estimate of the dredge performance is to be made and the economic operating conditions are to be determined.

Past investigation of this subject is very limited, including only brief reports by the Mississippi River Commission (1)[2]; by the Japanese investigators, E. Mikumo, T. Nishihara, and M. Takahara (2); by W. B. Gregory, M. Am. Soc. C. E. (3); and by M. P. O'Brien, M. Am. Soc. C. E., and R. G. Folsom (4). These investigations established the fact that pump characteristics are different for solid-water mixtures than for water alone, but since none of these reports include a complete description of the pumps used or of the material in suspension it is impossible to make any quantitative analysis of the observed change in characteristics.

The purpose of this investigation was to obtain the characteristics of a centrifugal pump of known dimensions while pumping in suspension material

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[1] Capt., Corps of Engrs., U. S. Army.

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (Appendix II).

of various known sizes and concentrations, and to make a quantitative analysis of the change in characteristics due to the material in suspension.

Experimental Tests

The system used in conducting the experimental tests was of a recirculating type as shown in Fig. 1. The pump impeller was 12 in. in diameter, with an

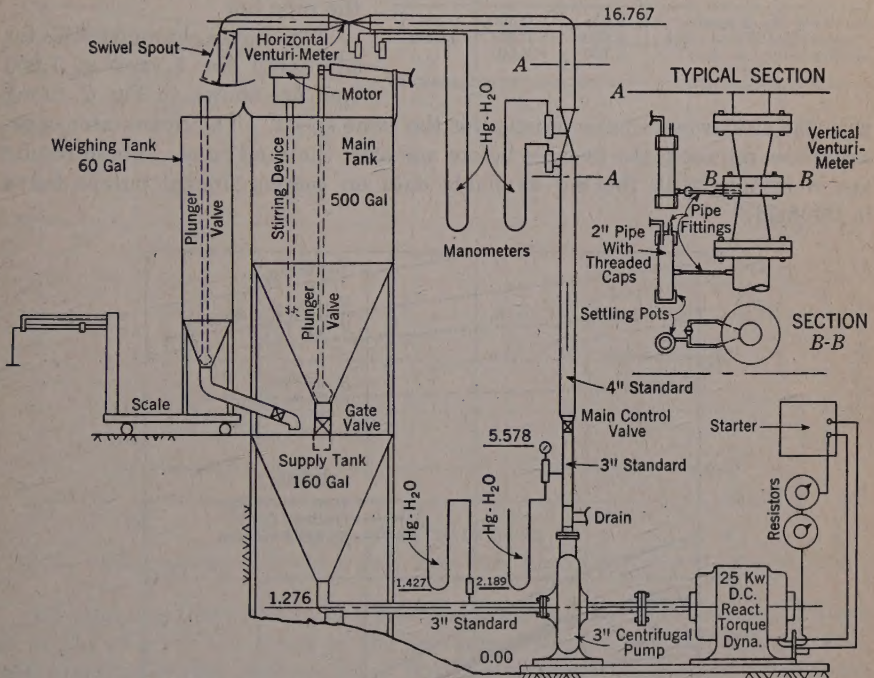

Fig. 1

inlet diameter of $5\frac{1}{4}$ in. and a width of $1\frac{3}{4}$ in. The shape of the impeller vanes closely approximated the form of a logarithmic spiral of the equation

$$r = e^{a\theta} \qquad (1)$$

in which the average value of a is 0.440, r is the radius, and θ is the central angle (see Appendix I).

The suspended material used in the tests included two grades of sand and one grade of oil-well drilling mud. Specific gravity and size distribution were determined for each material in accordance with American Society for Testing Materials (A. S. T. M.) standards. Average fall velocities for the two sands were determined by weighting observed fall velocities for fractions of each sand obtained from mechanical analysis (see Table 1).

Clear-water runs were made before and after pumping each different material in suspension, for the purpose of determining the effect of wear on

the pump characteristics. A complete series of runs was made at pump speeds of 1,000 and 1,200 rpm. The concentration of the suspended material was varied from 10% to 25% by net volume. Generally, the rates of discharge covered a range from 0.3 cu ft per sec to 1.0 cu ft per sec, although at the higher concentrations this lower limit could not be reached without clogging the pipe line.

TABLE 1.—Relative Fall Velocities

Material	Specific gravity	Median diameter, in mm	Fall velocity, in ft per sec
Monterey No. 4 sand.....	2.655	0.800	0.384
Crushed Del Monte sand..	2.630	0.034	0.0012
Mojave rotary mud.......	2.726	<0.001	....

The pump characteristics for Monterey No. 4 sand at 1,000 rpm are shown in Fig. 2, along with the clear-water characteristics for the same speed. The clear-water characteristics represent the average before and after the sand runs. These results are in keeping with the few available data on suction dredge pumps taken in the field.

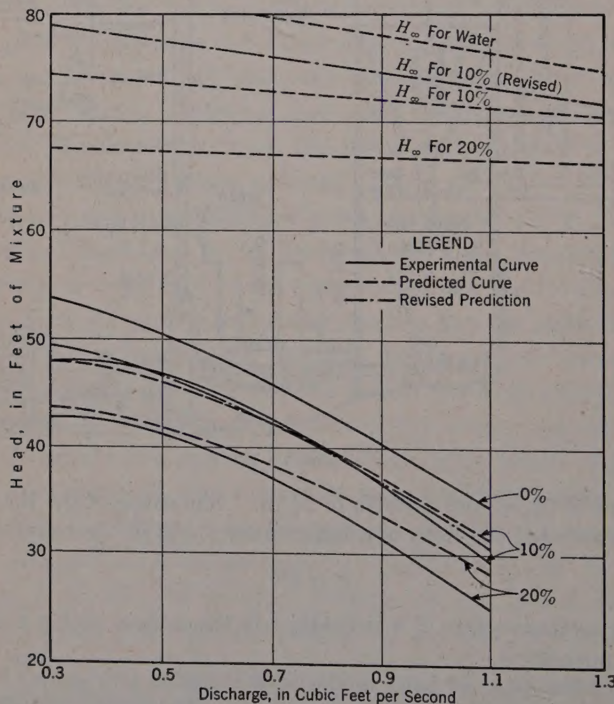

Fig. 2.—Comparison of Experimental and Predicted Characteristics with Monterey No. 4 Sand Pumped at 1,000 RPM

The head-capacity characteristics, expressed in feet of mixture, and the efficiency characteristics for the crushed Del Monte sand for all concentrations were found to coincide with the average clear-water characteristics. The ratio of power input to specific gravity of mixture also coincided with the

power input characteristic for clear water. The pump characteristics for the Mojave rotary mud at 1,000 rpm are shown in Fig. 3.

The usual affinity relations for rotational speeds ($Q \propto N$, $H \propto N^2$, and $H\,P \propto N^3$) were applied to the solid-water characteristics at 1,000 rpm, and results compared favorably with experimental characteristics at 1,200 rpm.

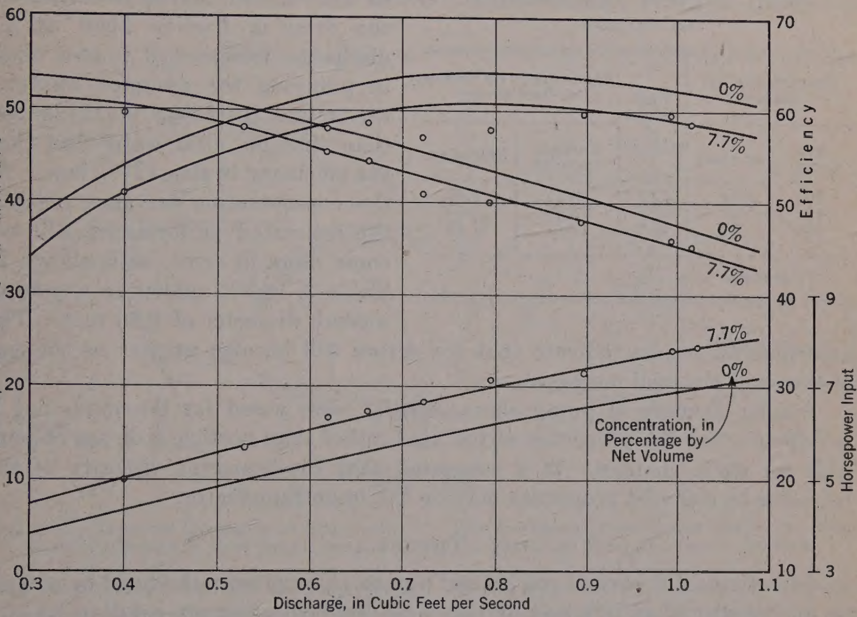

Fig. 3.—Characteristics of a 3-In. Dredge Pump with Mojave Rotary Mud, at 1,000 RPM

By applying these results to a practical problem some indication is afforded as to the effect of the errors in the assumptions commonly made in the field in estimating dredge performance. If a dredge pump is assumed of the same specific speed as the pump tested herein, the characteristics of the two pumps will be proportional. Assume a dredge pump of the following description:

Description	Dimension
Dimensions of Discharge Pipe, in Ft:	
Diameter	2
Length	1,950
Discharge velocity, in ft per sec	16.3
Pump speed, in rpm	212
Impeller diameter, in ft	6.7
Static lift, in ft	30
Total head, in ft	86

The foregoing conditions correspond to the conditions of maximum efficiency, both for the test pump at 1,000 rpm and the dredge pump at 212 rpm. Assume now that the dredge is pumping a mixture consisting of 15% by net volume of

Monterey No. 4 sand. (This is equivalent to a unit weight of mixture of 77.8 lb per cu ft, the upper limit reached in dredging operations in the field.) In ordinary practice the rate of discharge and efficiency of the dredge pump would be estimated as the same as for clear water. However, referring to Fig. 2, and taking into account the drop in friction head as the discharge decreases, it is seen that, in pumping the assumed mixture, the actual discharge is $12\frac{1}{2}\%$ less than that for clear water and that the efficiency is also $12\frac{1}{2}\%$ less. As the concentration becomes greater, the estimated performance will become more in error, as is shown in Table 2, which applies to a sand of median diameter of 0.80 mm. The experimental results indicate that the errors will become greater as the size of the solid material increases.

TABLE 2.—ESTIMATED PERFORMANCE WITH VARYING PERCENTAGES OF SOLIDS

PERCENTAGE OF SOLIDS		Unit weight of mixture	PERCENTAGE OF ERROR IN ESTIMATED:	
Net volume	In place		Yardage removed	Efficiency
10	14	72.5	6.25	9.38
15	21	77.8	12.50	12.50
20[a]	28	83.0	18.75	17.19

[a] Excessive.

Similar changes in pump characteristics were noted for the mud, but it is believed that the properties of the mud, other than particle size, are responsible for these changes. It is suggested that the apparent viscosity of the mud due to colloidal properties may be the important factor.

THEORY

The classical theory of centrifugal pumps (5) expresses the head developed by an impeller of an infinite number of blades, neglecting pre-rotation, as

$$H_\infty = \frac{U_2\, v_{u2}}{g} \qquad (2)$$

in which H_∞ is the head, in feet of fluid, U_2 is peripheral velocity of impeller, and v_{u2} is the tangential component of the absolute velocity at exit from the impeller. Eq. 2 gives the head in feet of fluid being pumped. To derive an expression for the actual head developed by a pump, it is necessary to include certain losses occurring both within the impeller and within the pump casing. Experiments indicate, however, that these losses are similar for both sand-water mixtures and clear water.

Referring to Eq. 2, it is seen that under conditions of constant speed the only manner in which the head developed could become less is that in which the value of v_{u2} also decreases. Since experiments indicate that for a sand-water mixture the head developed is less than that for water alone, at the same discharge, then the loss must be due to a decrease in the value of v_{u2} for the solid-water mixture, or, in other words, the solid particle must leave the impeller with a greater relative velocity than the water particles. This is shown graphically in Fig. 4.

From the geometry of the velocity diagrams at the exit from the impeller, in which $V_{f(2)}$ is the radial component of relative velocity with respect to

the impeller blade at the exit, and β is the blade angle:

$$v_{u2} = U_2 - V_{f(2)} \cot \beta \quad \text{.....................(3)}$$

Assume an impeller of an infinite number of blades and add to the assumptions made in the classical theory the assumption that the solid particles also travel a relative path that follows the contour of the vanes. Then an expression

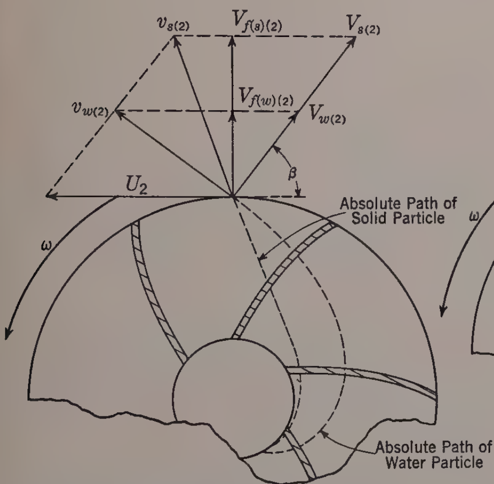

FIG. 4.—VELOCITY DIAGRAMS OF SOLID AND WATER PARTICLES AT EXIT FROM IMPELLER

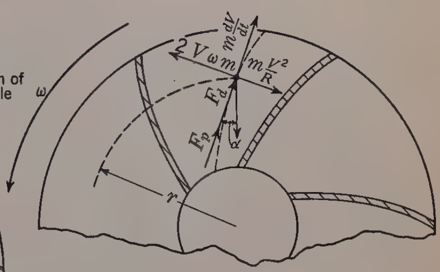

FIG. 5.—DIAGRAM OF FORCES ACTING ON SOLID PARTICLES PASSING THROUGH IMPELLER

can be obtained for the head developed by the impeller by adding the expressions for power imparted to the water and to the solid particles (which are determined from the time rate of change of moment of momentum), equating this sum to the expression for total power (which equals the product of the discharge per unit time, the unit weight of the mixture, and the head developed in feet of mixture), and then solving for the value of this head. Performing the foregoing operations, the expression for the theoretical head is obtained, expressed in feet of mixture,

$$H_\infty = \frac{S_s\, p_s\, U_2 \,(U_2 - V_{f(s)(2)} \cot \beta) + (1 - p_s)\, U_2 \,(U_2 - V_{f(w)(2)} \cot \beta)}{g\, S_a} \quad .(4)$$

In Eq. 4, subscript s refers to solid particles, w to water particles, 2 to the exit from the impeller, a to the average of solid-water mixture, S is specific gravity, and p is concentration of solids in percentage by volume.

The solution of Eq. 4 requires the value of $V_{f(w)(2)}$, which can be determined by the continuity relationship on the basis that the impeller is completely filled with water, and the value of $V_{f(s)(2)}$, which is not so readily determined.

Referring to Fig. 5, which shows the forces acting on a solid particle in a fluid at any position within the impeller, it is seen that there are six forces acting. If the forces acting tangent to the impeller vane are equated:

$$m \frac{dV_s}{dt} = m\, \omega^2\, r \cos \alpha - F_d - F_p \quad \text{..................(5)}$$

in which m is the mass of a solid particle and ω is the angular velocity of the impeller. By assuming that F_d is a viscous force, that F_p is the force due to the pressure gradient in the fluid surrounding the particle, and that the impeller vane approximates a logarithmic spiral, and by taking an average value of V_w as it appears in the expressions for F_d and F_p, it is possible to obtain the following equations describing the motion of the solid particle:

$$r = C_1\,e^{Xt} + C_2\,e^{Yt} - \frac{F}{B} \qquad \text{(6a)}$$

and

$$V_s = \frac{\sqrt{1 + a^2}}{a}\,(C_1\,X\,e^{Xt} + C_2\,Y\,e^{Yt}) \qquad \text{(6b)}$$

in which C_1 and C_2 are constants of integration, and X, Y, F, and B are constants depending on particle size, settling velocity, and operating conditions of the pump (see Appendix I, Eqs. 11). By substituting the boundary conditions at the entrance to, and exit from, the impeller, the value of $V_{s(2)}$ can be determined, making possible the solution of Eq. 4.

It has been found, from a consideration of the equations, that, as the size of the solid particle, and hence its fall velocity, become smaller, the difference between V_s and V_w becomes less, so that, for very small solid particles (such as crushed Del Monte sand) in suspension, the head developed will be the same as for water alone.

The predictions of the operating characteristics while pumping a suspension may be obtained by expressing the difference between H_∞ as computed by Eq. 4 and H_∞ for clear water as a percentage loss, and applying this loss to the actual clear-water characteristics of the pump.

A comparison between predicted and test characteristics for Monterey No. 4 sand at 1,000 rpm is given in Fig. 6. For the rating point of the pump the predicted characteristic is correct for a concentration of 10% by net volume and is within 5% of the test value for a concentration of 20%. This same agreement was found to hold true at 1,200 rpm. A sample computation for this prediction is given in Appendix I.

The errors introduced through the assumptions made in this discussion are well appreciated. For the most part they are the same as those made in the classical theory, which are well known to be in error and for which it is possible to make corrections. Probably the least valid of the assumptions, at least when the suspended material has a high settling velocity, is that as to the nature of the resistance force between the solid and water particles. It is a well-known fact that Stokes' law is valid only in the range of Reynolds' number less than 0.50, and, since this value is probably greatly exceeded in most cases, it would be more nearly correct to use Rittenger's or Allen's law (6). However, the application of such a law involves the solution of a complicated differential equation. For the pump and suspended material used in the experimental tests, values of H_∞ were computed by a trial-and-error method, using Rittenger's law. These values were about 7% higher than those determined by the less accurate solution. However, if the difference between H_∞ for water and H_∞ for the suspension, computed by trial and error,

is applied directly as a loss to the actual clear-water characteristics, the predicted characteristics for the suspension agree within 4% with the experimental characteristics. This agreement is shown in Fig. 6. In view of the increased

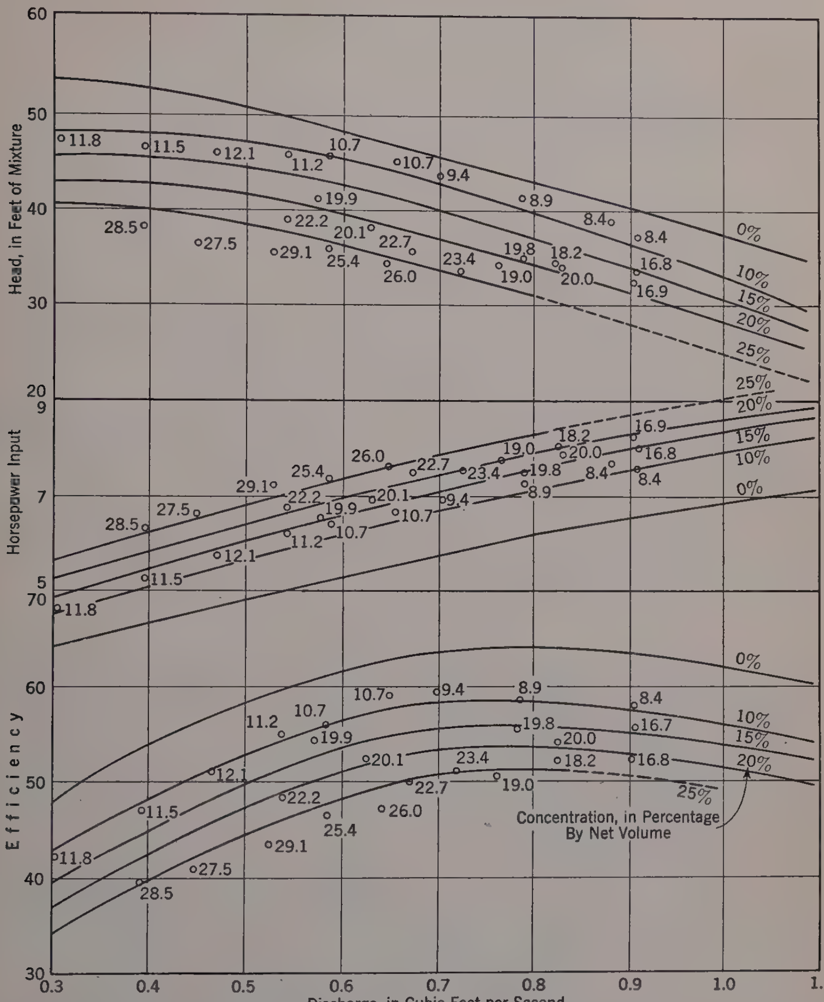

FIG. 6.—CHARACTERISTICS OF A 3-IN. DREDGE PUMP WITH MONTEREY NO. 4 SAND, AT 1,000 RPM

difficulty and expenditure of time in making the trial-and-error solution, the method first described is the more practical, especially since both methods are subject to several approximations.

CONCLUSIONS

The following conclusions are based on the observations made in this investigation:

(1) At a given capacity the head developed, in feet of mixture, by a centrifugal pump handling material in suspension, is less than that developed for water alone;

(2) The drop in the head-capacity characteristics at constant speed varies not only as the concentration but also as the particle size of the material in suspension;

(3) The fall velocity of the suspended material is the most important property in predicting the effect of the material on the pump performance;

(4) The effect on the pump characteristics of very fine particles in suspension, such as muds having colloidal properties, is of a different nature than that of a true suspension;

(5) The power input to a centrifugal pump varies directly with the apparent specific gravity of the suspension being pumped;

(6) The capacity at maximum efficiency of a centrifugal pump remains about constant for various concentrations and sizes of suspended materials;

(7) The ordinary affinity relations of centrifugal pumps are valid within small ranges of speed when pumping material in suspension;

(8) The theoretical analysis made herein indicates the nature of the changes in the head-capacity characteristics of a centrifugal pump handling solid materials in suspension, and will serve as a first approximation in predicting pump performance; and

(9) Suspended material, of sufficient size and concentration to change pump characteristics, makes the present method of estimating pump performance from clear-water tests definitely in error.

Acknowledgment

The experimental study described in this paper was made by the author in the Fluid Mechanics Laboratory of the University of California, in Berkeley, Calif. It was reported completely in a thesis by the author, entitled "The Effect of Material in Suspension Upon the Characteristics of a Centrifugal Pump," and presented to the University in 1940 in partial fulfilment of the requirements for the degree of Master of Science.

APPENDIX I

Sample Computations

In order to make a sample computation it will be necessary to review briefly the steps in deriving Eq. 5 so as to determine the constants X, Y, F, and B. The term F_p which appears in Eq. 5 will be neglected since, for the particular material in suspension, this term affected the solution of that formula only 0.5%. From the assumption that F_d is a viscous force, Stokes' law states

$$F_d = K (V_s - V_w) \qquad (7)$$

From the assumption that the impeller vane is a logarithmic spiral: $r = e^{a\theta}$

(Eq. 1); $\cos \alpha$ = a constant; and

$$V_s = \frac{dr}{dt} \frac{\sqrt{1 + a^2}}{a} \qquad (8)$$

By taking an average value of V_w as it appears in Stokes' law, Eq. 5 may be written:

$$\frac{d^2 r}{dt^2} = \frac{a\, \omega^2\, r \cos \alpha}{\sqrt{1 + a^2}} - \frac{K}{m}\frac{dr}{dt} + \frac{a\, K\, V_{(w)\,\text{average}}}{m\sqrt{1 + a^2}} \qquad (9)$$

Let:

$$A = \frac{K}{m} \qquad (10a)$$

$$B = -\frac{\omega^2\, a \cos \alpha}{\sqrt{1 + a^2}} \qquad (10b)$$

and

$$F = -\frac{a\, K\, V_{(w)\,\text{average}}}{m\sqrt{1 + a^2}} \qquad (10c)$$

Substituting Eqs. 10 in Eq. 9:

$$r = C_1\, e^{Xt} + C_2\, e^{Yt} - \frac{F}{B} \qquad (6a)$$

and

$$V_s = \frac{\sqrt{1 + a^2}}{a}\left(C_1\, X\, e^{Xt} + C_2\, Y\, e^{Yt}\right) \qquad (6b)$$

in which (to simplify typography):

$$X = \tfrac{1}{2}\left(-A + \sqrt{A^2 - 4B}\right) \qquad (11a)$$

and

$$Y = \tfrac{1}{2}\left(-A - \sqrt{A^2 - 4B}\right) \qquad (11b)$$

The physical properties of the pump are:

a = 0.44 (the constant in equation of a logarithmic spiral);
r_1 = 0.167 ft (inlet radius of impeller);
r_2 = 0.500 ft (exit radius of impeller);
b_1 = 0.183 ft (width of impeller passage at inlet);
b_2 = 0.146 ft (width of impeller passage at exit);
A_1 = 0.192 sq ft (area of impeller passage at inlet);
A_2 = 0.459 sq ft (area of impeller passage at exit);
α = 66° 16′ (from the equation of the blade; the acute angle between the tangent to the blade and the radius of the impeller);
β = 23° 44′ (from the equation of the blade; blade angle at 1,000 rpm);
ω = 104.72 radians per sec (angular velocity of impeller); and
U_2 = 52.36 ft per sec (peripheral velocity of impeller).

The physical properties of Monterey No. 4 sand are: Median diameter, D = 0.800 mm; fall velocity, v = 0.384 ft per sec; and specific gravity, S_s

= 2.655. From Stokes' law—

$$K\,v = \frac{\pi\,D^3}{6}\,(S_s - 1)\,w \qquad (12)$$

Also, the mass of a solid particle is

$$m = \frac{\pi\,D^3\,S_s\,w}{6\,g} \qquad (13)$$

and $\frac{K}{m} = 52.271$.

Assume a pump speed of 1,000 rpm, a discharge of 1 cu ft per sec, and a concentration of sand of 10% by net volume. Substituting in Eqs. 10 and 11: $A = \frac{K}{m} = 52.271$; $B = -1{,}776.603$; $F = -174.188$; $\frac{F}{B} = 0.098$; $X = 23.460$; and $Y = -75.731$.

When $r = r_1 = 0.167$ ft, $t = 0$ and $V_{s(1)} = \frac{Q}{A_1}\sin\beta = 12.956$ ft per sec. Substituting in Eq. 6*a*:

$$0.167 = C_1 + C_2 - 0.098 \qquad (14a)$$

Substituting in Eq. 6*b*:

$$5.221 = 23.460\,C_1 - 75.731\,C_2 \qquad (14b)$$

Solving for C_1 and C_2: $C_1 = 0.255$ and $C_2 = 0.010$.

When $r = r_2 = 0.500$ ft, substitute in Eq. 6*a*: $0.500 = 0.255\,e^{23.460t} + 0.010\,e^{-75.731t} - 0.098$; and $t_2 = 0.0362$ sec. Substituting in Eq. 6*b*: $V_{s(2)} = 34.536$ ft per sec. Furthermore, $V_{f(s)(2)} = V_{s(2)}\sin\beta = 13.928$ ft per sec; and $V_{f(w)(2)} = \frac{Q_w}{A_2} = 1.961$ ft per sec. Substituting in Eq. 4: $H_\infty = 67.809$ ft, for Monterey No. 4 sand, at 1,000 rpm, 1 cu ft per sec, and 10% concentration. Repeating the foregoing steps for the same conditions but for a discharge of 0.3 cu ft per sec, $H_\infty = 73.195$ ft.

It is to be noted that the foregoing results vary with those shown in Fig. 6 but give a much better agreement between the predicted and actual characteristics.

APPENDIX II

Bibliography

(1) **"Pipe Line Dredges,"** by E. J. Dent, *Professional Memoirs*, Corps of Engrs., U. S. Army, Vol. VII, 1915, No. 31.

(2) **"On the Hydraulic Conveyance of Slime Pulps (Copper Ore),"** by E. Mikumo, T. Nishihara, and M. Takahara, *Journal*, Soc. of Mech. Engrs., Japan, Vol. XXXVI, December, 1933, pp. 825–831.

(3) **"Pumping Clay Slurry through a Four-Inch Pipe,"** by W. B. Gregory, *Mechanical Engineering*, Vol. 49, 1927, pp. 609–616.

(4) **"The Transportation of Sand in Pipe Lines,"** by M. P. O'Brien and R. G. Folsom, *Publications in Engineering*, Univ. of California, Vol. 3, No. 7, 1937, pp. 343–384.

(5) **"Centrifugal Pumps,"** by R. L. Daugherty, McGraw-Hill Book Co., Inc., New York, N. Y., 1915.

(6) **"The Laws of Motion of Particles in a Fluid,"** by R. G. Lunnon, *Transactions*, Institution of Mining Engrs., Vol. LXXVII, 1929, pp. 65–74.

(7) **"A Study of Some Phases of Chemical Control in Clay Suspensions,"** by A. D. Garrison and K. C. Ten Brink, *Petroleum Technology*, Am. Inst. of Mining and Metallurgical Engrs., Vol. 2, No. 4, 1939.

MECHANICS OF FLOW, WITH NONCOLLOIDAL, INERT SOLIDS

By Warren E. Wilson, Assoc. M. Am. Soc. C. E.[3]

Synopsis

The investigation of the laws governing the pipe-line flow of liquids containing solids in suspension has received the attention of a number of investigators in several fields. The presentation and analysis of the data in most cases have been of such a nature as to provide essentially an immediate answer to questions concerning the efficiency of the pumping process. Little progress has been made toward the unification of the data by means of a comprehensive theoretical background.

In the field of the transportation of solids by streams in open channels notable work has been done by a number of men, prominent among whom are Morrough P. O'Brien and E. W. Lane, Members, Am. Soc. C. E., and J. E. Christiansen, A. A. Kalinske, and Hunter Rouse, Assoc. Members, Am. Soc. C. E., to provide a clearer understanding of the mechanics of the turbulence process in maintaining material in suspension. However, a direct application of their approach to the problem of the transportation of solids in pipes does not seem feasible at the present time.

The purpose of this study is to provide an elementary theory on the basis of which certain features of the available experimental data on the transportation of solids in pipes may be interpreted. The theory is neither complete nor directly concerned with the details of the mechanics of the turbulence process in maintaining solids in suspension, but rather represents an analysis of the mechanics of the flow on the basis of energy relationships; and it results in an explanation of several important and previously noted but unsatisfactorily explained features of the flow.

Experimental data from several sources are presented and analyzed on the basis of the background provided by the theory. Due to lack of complete data in many old studies of flow of the type under consideration it is impossible to make use of the results of much of the experimental work.

Theory

The analysis will be limited to the consideration of the transportation of noncolloidal, inert solids for which the average velocity of settling relative to the transporting liquid is known. Complicating factors such as chemical re-

[3] Prof. and Head, Dept. of Mechanics, Colorado School of Mines, Golden, Colo.

action between solids and liquid and the formation of semi-solid plugs, as noted by some observers, will be assumed negligible or nonexistent.

Consider a liquid flowing in a horizontal pipe, as illustrated in Fig. 7, toward the right at the constant average velocity V. Particles such as P are in suspension. If there were no turbulence the particle P would settle at the velocity V_s and in the period of time T would move downstream the distance $L = V\,T$ and vertically downward the distance $V_s\,T$. The flow being steady the particles will maintain their average positions in the vertical relative to the pipe wall. Now, since the particles individually do settle and are raised up to new positions by the turbulence of the liquid, maintaining the same average position, it is clear that the liquid must do work on the particles definitely related to the decrease in potential energy due to settling. It is not to be expected that the work done is exactly equal to this change in potential energy since the motion of the particles in settling relative to the liquid is accompanied by a loss in mechanical energy of the particles due to the viscosity of the liquid. Similarly additional energy will be required in raising the particles due to the viscous resistance requiring that more work be done than is necessary to raise the particles vertically a given distance.

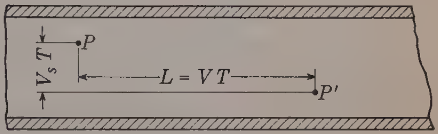

Fig. 7

In order to express this relationship between work done by the liquid on the particles in maintaining the suspension, and the decrease in potential energy during settling, one may write:

$$K\,p\,A\,L\,w\,V_s\,T = w\,h_1\,A\,L \quad \text{(15)}$$

in which: A is the cross sectional area of the flow section; p is the weight of solids per unit weight of solid-liquid mixture; w is the unit weight of the mixture; K is a coefficient associated with the efficiency of the process; and h_1 is the decrease in the elevation of the energy grade line in the length L due to the maintenance of the solids in suspension. Now recalling that

$$L = V\,T \quad \text{(16)}$$

Eq. 15 may be simplified into the form,

$$\frac{h_1}{L} = \frac{K\,p\,V_s}{V} \quad \text{(17)}$$

which is a very simple expression for the slope of the energy gradient due to the maintenance of the solids in suspension.

The Weisbach-Darcy equation may be used to determine the slope of the energy gradient due to the resistance of a homogeneous liquid flowing in a given pipe, and it will be assumed that this equation will apply equally well to a non-homogeneous liquid-solid mixture. Therefore,

$$\frac{h_2}{L} = \frac{f\,V^2}{2\,g\,D} \quad \text{(18)}$$

in which: f is the friction factor; D is the pipe diameter; $\frac{h_2}{L}$ is the slope of the energy gradient due to the flow of the liquid mixture expressed in feet of mixture per foot of pipe; and g is the acceleration due to gravity.

An expression for total energy gradient follows immediately from the addition of Eqs. 17 and 18—

$$\frac{H}{L} = \frac{f\,V^2}{2\,g\,D} + \frac{K\,p\,V_s}{V} \qquad (19)$$

Eq. 19 expresses the fundamental principle of this study, the concept that the total energy gradient may be divided into two parts—the first, that which expresses the rate at which potential energy is being dissipated to maintain the turbulent flow of a liquid in a pipe, and the second, that which expresses the rate at which energy is being supplied to maintain the solids in suspension.

It is immediately apparent that for a given pipe, percentage of solids, and particle size and shape which are characterized by V_s, the total energy gradient approaches that for a clear liquid as V increases, since the second term of Eq. 19 decreases as V increases and the first term is the energy gradient for a clear liquid expressed in terms of head of the mixture per unit length of pipe. This phenomenon has been repeatedly observed by investigators in this field.

It is evident also that for a given solid-liquid mixture and size of pipe there will be a velocity that gives a lesser energy gradient than any other velocity. The approximate magnitude of this velocity may be determined by assuming p, V_s, f, K, and D constant, then differentiating the expression for $\frac{H}{L}$ with respect to V and setting the first derivative equal to zero. The resulting criterion for the critical value of the velocity is,

$$V^3_c = \frac{K\,p\,V_s\,g\,D}{f} \qquad (20)$$

in which V_c is the critical velocity, giving minimum energy gradient for the given combination of pipe and mixture.

Thus far it has been assumed that all the solids remain in suspension. However, it is apparent and a matter of common observation that under certain circumstances the solids will settle out, and eventually clog the pipe. The criterion for settling out of the particles or the picking-up of particles from the bed has been discussed by Messrs. Lane and Kalinske.[4] These writers demonstrate that, if the quantity $\frac{V'_s}{\sqrt{\frac{\tau_0}{\rho}}}$ (in which V'_s is the settling velocity of the solids, τ_0 is the bottom shearing stress per unit area, and ρ is the unit density of the liquid) is equal to or greater than unity little or none of the material with settling velocity V'_s will be placed in suspension.

[4] "Relation of Suspended to Bed Material in Rivers," by E. W. Lane and A. A. Kalinske, *Transactions*, Am. Geophysical Union, 1939, p. 637.

Assuming that the shearing stress τ_0 is equal for all parts of the wetted perimeter it follows that, for an element of the liquid, of length L and diameter D_l, the equation of equilibrium in the direction of flow is:

$$(p_1 - p_2)\frac{\pi D^2_l}{4} - \tau_0 \pi D_l L = 0 \qquad (21)$$

in which p_1 and p_2 are the unit pressures at the upstream and downstream ends of the element, respectively. If H is the decrease in pressure head in the length L, it follows that

$$\frac{\tau_0}{\rho} = \frac{H g D_l}{4 L} \qquad (22a)$$

or upon substitution of the hydraulic radius for $0.25\, D_l$:

$$\frac{\tau_0}{\rho} = \frac{H g R}{L} \qquad (22b)$$

which will give for the quantity $\sqrt{\frac{\tau_0}{\rho}}$ the expression,

$$\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{H g R}{L}} \qquad (23)$$

Thus, the condition for the settlement of solids becomes,

$$\frac{V'_s}{\sqrt{\frac{H g R}{L}}} = 1 \qquad (24)$$

The assumption of uniform shear stress around the wetted perimeter is not likely to be fulfilled and there will be settling out of solids at values of the ratio of Eq. 24 corresponding to velocities somewhat different from those indicated by a value of unity of the ratio.

Although some data are available concerning the minimum velocities necessary to prevent clogging the auxiliary data are incomplete and afford no basis for checking the validity of Eq. 24.

A means of determining the suitability of Eq. 24 as a criterion for deposition of solids was made possible by the observations on the flow in the tailings disposal line of the Climax Molybdenum plant at Climax, Colo. The important characteristics of the situation that exists in this pipe line are depicted in Fig. 8. Since the flow in this line represents a type entirely different from that in pipe which is commonly supplied by a pump, it merits consideration under this

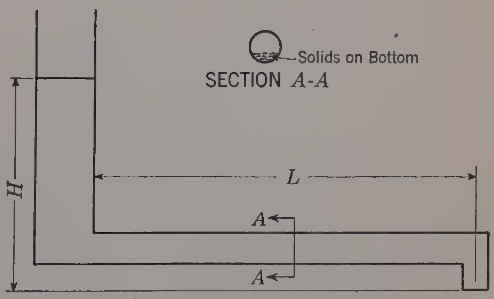

FIG. 8

section, setting forth the elements of the mechanics of the flow of solids suspended in liquids in pipe lines.

Fig. 8 illustrates a horizontal pipe fed from a vertical stand-pipe open to the atmosphere, and discharging from a right-angled bend, vertically downward. The elevation of the free surface in the stand-pipe is determined by the loss of head in the pipe line. This loss, in turn, is dependent upon all the factors indicated by Eq. 19. It is apparent that if the discharge, percentage of solids, or type or size of particles changes the head loss will change. An automatic adjustment of the head takes place in the case of any of these changes and the flow eventually becomes steady. It is apparent that if, with a given discharge, the velocity is so low that solid particles are deposited on the bottom, this deposition will continue until the cross section has become obstructed to such an extent that the velocity is high enough to maintain the suspension, and the pipe will not clog since the head will automatically adjust itself to the new steady state. Such is found to be the mode of operation of this pipe line. Obviously a pipe supplied from a pump that maintains a constant head could not operate in this manner since the automatic head adjustment would not be possible.

Thus far, a horizontal pipe has been assumed. If the slope of the pipe is very large the component of the settling velocity perpendicular to the pipe wall must be considered as the quantity V_s rather than the full vertical velocity with which the solids settle relative to the liquid. In the case of small slopes this factor will be of minor importance but will doubtless be a factor of considerable importance if the slope is very large. If the pipe is vertical, obviously there is no component of the settling velocity perpendicular to the pipe wall; hence, in this case, one would expect no increase in the energy gradient due to the maintenance of the suspension.

Description of Experimental Data

To investigate the applicability of the foregoing theory, three sets of experimental data were used. Several others were investigated but were found to be lacking in some essential feature. The fact that it is necessary to know the settling velocity of the particles, as well as the percentage of solid material in suspension, eliminates from consideration much data that are available since a screen analysis of the material used is seldom available.

The first data studied were obtained by Nora Stanton Blatch and published in 1906.[5] Miss Blatch studied in detail the flow of sand-water mixtures in 1-in. pipe. Both percentage of solids and screen analysis are available in these data.

A second set of experimental results by G. W. Howard, Jun. Am. Soc. C. E., published in 1939,[6] contains data on the flow of sand-water and gravel-water mixtures in 4-in. pipe. These data are complete with screen analyses.

A third set of data (see Table 3) was supplied by the Climax (Colo.) Molybdenum Company and is complete in practically every detail. This material describes the flow of tailings in an 18-in. wood-stave pipe line so con-

[5] *Transactions*, Am. Soc. C. E., Vol. LVII, December, 1906, pp. 400–408.

[6] *Loc. cit.*, Vol. 104 (1939), pp. 1334–1348; also, *loc. cit.*, Vol. 106 (1941), pp. 135–157.

structed and connected as to give a unique flow of the mixture differing in several important respects from the flow in the conventional pipe line setup.

The essential data are introduced in the following sections when necessary for the discussion. The settling velocity (Table 3) in all cases was estimated on the basis of the grain size only and no allowance was made for shape of particles; hence, the data may well be in error to some extent. However, this fact will not materially detract from the value of the conclusions drawn.

Analysis of Data

Fig. 9 is a plot of the experimental data assembled by Miss Blatch. The head loss in a 1-in. pipe per unit length, expressed in terms of the head of the mixture, is plotted as the ordinate and the average velocity of the mixture in the pipe is plotted as the abscissa. Each curve represents the data for the flow with a given average settling velocity of particles in the mixture and a given percentage of solids.

Sands designated as Nos. 1 and 2, respectively, were used in the work done by Miss Blatch. All of sand No. 1 passed the 20-mesh and was retained on the 40-mesh sieve. It is estimated that the average settling velocity of this material in water was 0.213 ft per sec. Sand No. 2 all passed the 60-mesh and was retained on the 100-mesh

TABLE 3.—Flow Data, Tailing Line, Climax Molybdenum Company, Climax, Colo.

Run No.	Flow (cu ft per min)	Solids		% plus 100 mesh	Hydraulic gradient, S	Velocity[a] (ft per sec)	Open cross section, %
		%	Dry tons in 24 hr				
(*a*) Series 1 (Length, 1,900 Ft; Slope, 0.003; Wood-Stave Pipe, Inside Diameter, 18 In.; Fed by Penstock 3 Ft by 4 Ft [Inside], 25 Ft High)							
1	412	34.3	7,890	20.0	0.0069	5.1	77[c]
2	402	39.4	9,480	25.6	0.0074	5.15	76[c]
3	406	41.4	10,340	31.3	0.0076	5.1	76[c]
4	404	44.0	11,080	34.8	0.0078	5.1	75[c]
5	409	49.4	12,920	40.8	0.0089	5.1	76[c]
6	405	51.1	13,740	43.4	0.0092	5.15	74[c]
7	402[b]	55.8	15,600	47.8	....	....	....
8	206	34.3	3,945	20.0	0.0063	4.35	45[c]
9	201	39.4	4,740	25.6	0.0070	4.35	44[c]
10	203	41.4	5,170	31.3	0.00765	4.25	45[c]
11	202	44.0	5,540	34.8	0.0083	4.33	44[c]
12	205	49.4	6,460	40.8	0.0090	4.30	45[c]
13	202	51.1	6,870	43.4	0.0100	4.35	44[c]
(*b*) Series 2; East Line, Station 4400 to Station 6174 (Length 1,774 Ft; Slope 0.00384; Penstock at Station 4400, of Circular Tile, Inside Diameter 30 In.)							
1	426	36.1	8,250	21.7	0.0080	5.9	68
2	402	40.8	9,120	25.6	0.0089	5.9	64
3	415	43.1	10,140	29.8	0.0083	5.65	69
4	399	44.7	10,350	33.6	0.0087	5.9	63
5	408	51.1	12,600	40.0	0.0090	5.7	67
6	405	53.1	13,290	43.4	0.0102	5.9	64
7	412	58.3	15,600	47.8	0.0100	6.6	59
8	213	36.1	4,125	21.7	0.0067	4.6	44
9	201	40.8	4,560	25.6	0.0076	4.6	42
10	208	43.1	5,070	29.8	0.0083	4.4	44
11	200	44.7	5,175	33.6	0.0092	4.1	40
12	202	51.1	6,300	40.0	0.0092	4.4	42
13	202	53.1	6,695	43.4	0.0112	4.6	42
(*c*) Series 2; East Line, Station 4400 to Station 5911 (Length, 1,511 Ft; Slope, 0.00384)							
14	260	37.8	5,400	22.9	0.0084	4.9	50
15	240	42.9	5,820	26.4	0.0077	5.0	47
16	294	42.7	6,720	31.0	0.0106	5.1	55
17	297	47.9	8,460	36.9	0.0098	4.8	59
18	297	51.6	9,330	39.5	Lost	4.8	58
19	267	57.2	9,510	46.7	0.0123	4.8	52
(*d*) Series 2; West Line, Station 5800 to Station 7700 (Length, 1,900 Ft; Slope 0.003; Penstock, 3 Ft by 4 Ft Wood Crib)							
20	412	34.9	7,560	20.0	0.0060	5.1	77
21	406	42.8	10,020	31.3	0.0077	5.1	76
22	404	46.0	10,860	34.8	0.0079	5.1	75
23	425	47.2	11,820	34.1	0.0077	5.3	76
24	409	51.2	12,540	40.8	0.0091	5.1	76
25	430	49.2	12,060	38.3	0.0085	4.9	83
26	207	43.1	5,070	29.8	0.0087	4.2	46
27	200	44.7	5,175	33.6	0.0092	4.1	46
28	212	47.2	5,910	34.1	0.0074	4.4	46
29	215	49.2	6,030	38.3	0.0088	4.7	43
30	202	51.1	6,300	40.0	0.0094	4.4	44

[a] Velocity determined by adding salt to the intake of the penstock, analyzing the pipe discharge for chlorine and noting the time for salt to appear at the discharge end.

[b] The tailing on this grind and density surged in the penstock; evidence of a tendency to become choked.

[c] Average free area; the part of the pipe area required to carry the given volume at a determined velocity.

sieve. It is estimated that the average settling velocity of this material in water was 0.0655 ft per sec.

In Fig. 9 it is easily seen that the important characteristics of the flow of the sand-water mixtures are in essential agreement with the predictions of the foregoing theory. Each curve, representing individually a given mixture, shows the characteristic minimum energy gradient at a certain velocity, with the exception of the curve for $V_s = 0.21$ ft per sec and $p = 0.12$ which does not clearly show such a minimum. Although the curves do not all become asymptotic to the curve for clear water, and in some cases drop below the clear-water curve, the general tendency is as predicted and the explanation for the

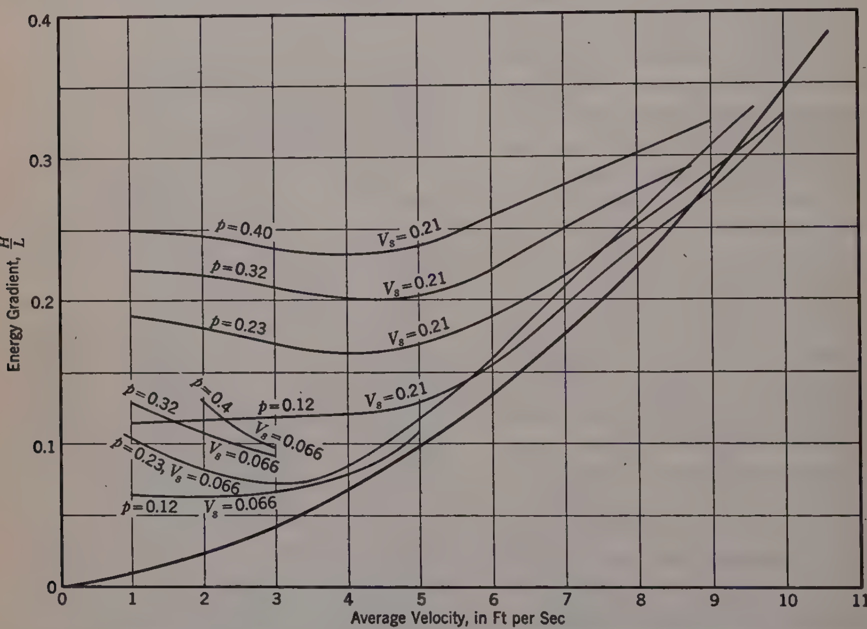

FIG. 9.—ENERGY GRADIENTS FOR THE FLOW OF WATER-SAND MIXTURES IN ONE-INCH PIPE

dropping below the clear-water curve is doubtless that the presence of the solids modifies the characteristics of the flow to such an extent that the energy gradient for the mixture is no longer comparable to that for clear water. The presence of the solids has been thought to act as a damping influence on the turbulent fluctuations of the velocity thus decreasing the energy gradient.

The experimental work was apparently not extended to the point at which it would be possible to determine the clogging velocities for the various mixtures. Therefore, it is impossible to learn anything concerning the applicability of the criterion for settling of the solids.

It is important to note that, in general, the head loss is a function of velocity at low velocities, and as had been observed in some experimental work independent of velocity over only small ranges of velocity. The variation of the head loss with velocity in the low velocity range is small in some cases.

Fig. 10 is similar to that of Fig. 9 and depicts the data obtained by Mr. Howard, who used a 4-in. pipe and a sand for which he presented a screen analysis. On the basis of the screen analysis it is estimated that the average settling velocity of the sand in water is 0.16 ft per sec.

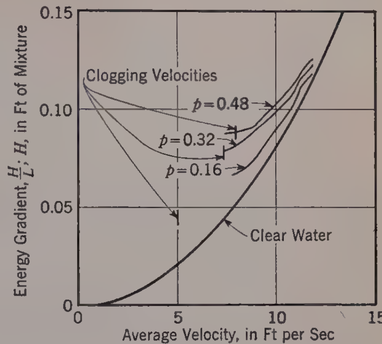

FIG. 10.—ENERGY GRADIENTS FOR THE FLOW OF WATER-SAND MIXTURES IN FOUR-INCH PIPE (AVERAGE SETTLING VELOCITY, V_s = 0.16 FT PER SEC)

The same general characteristics which were indicated by Miss Blatch's data are discernible in Mr. Howard's work, but the fact that the data cover a relatively narrow velocity range precludes the exact determination of the velocity for minimum head loss. The clogging velocities, experimentally determined by Mr. Howard, are indicated on the diagram by the short vertical lines so marked.

Fig. 11 shows a graphical representation of the data observed at Climax. The slope of the energy gradient is plotted as the ordinate and the quantity $\frac{p\,V_s}{V}$ as abscissa. The points are grouped according to approximately equal rates of discharge since the data were gathered by maintaining nearly constant rates of discharge with varying solids content in each series of runs. The data cover so narrow a range of velocities that a plotting of the type shown in Figs. 9 and 10 is not possible.

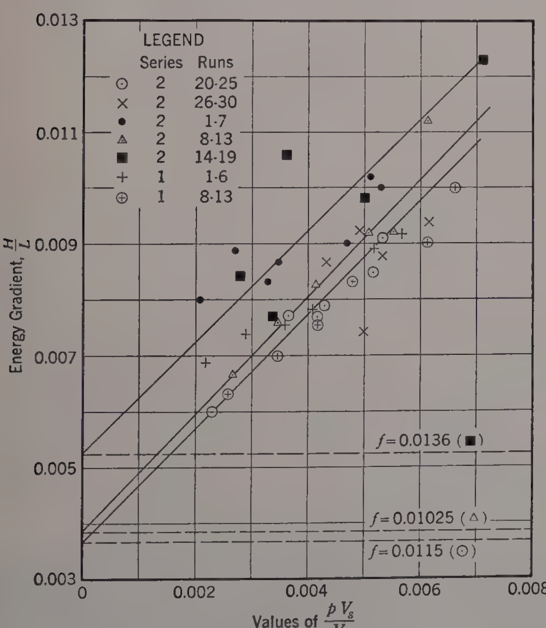

FIG. 11.—ENERGY GRADIENTS FOR THE FLOW OF TAILINGS IN 18-IN. WOOD-STAVE PIPE (f COMPUTED FOR A GIVEN DISCHARGE AND APPARENT $\frac{H}{L}$ FOR ZERO SOLIDS CONTENT)

In each series of runs both the percentage and size of solids were varied. Corresponding changes in the slope of the energy grade line took place. These data are summarized in Table 3. It is interesting to note that for a given discharge and with varying percentages and sizes of solids, the average velocity in the cross section remains nearly constant. In other words,

the variation in the quantity and average size of the solids had a minor effect upon the amount of deposition of material. This is in accordance with the conditions for the settling out of the solids.

The criterion for the deposition of solids from the suspension is expressed as

$$\frac{V_s}{\left(\frac{g\,r\,H}{L}\right)^{0.5}} = 1 \qquad (25)$$

It is apparent that aside from the effect on the energy gradient, the percentage of solids does not enter into the consideration. The settling velocity V_s is that of the largest particles present in sufficient quantity to be a factor in the clogging of the pipe since these large solids would settle first. Evidently, then, a change in the average size of the particles need not greatly affect this maxi-

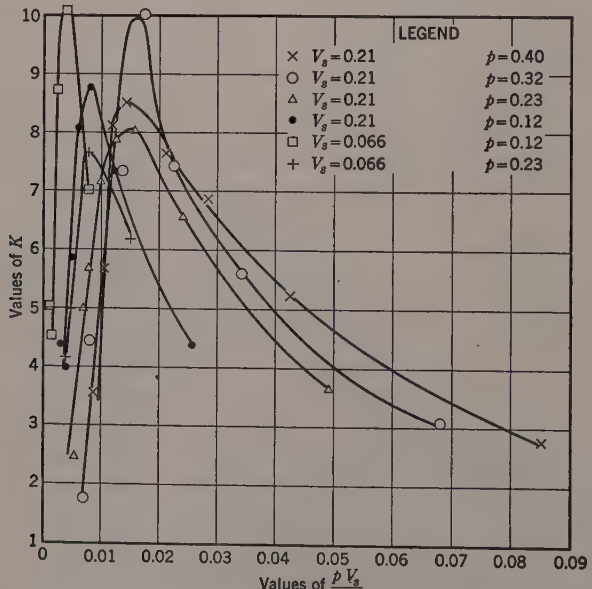

FIG. 12.—COEFFICIENT K AS A FUNCTION OF THE QUANTITY $\frac{p\,V_s}{V}$

mum settling velocity. The fact was observed that the open area of the cross section and average velocity changed little for a given discharge when the solids content was varied over a wide range. This phenomenon is explained satisfactorily on the basis of a slight change in the maximum settling velocity and an increase in the energy gradient sufficient in amount to compensate for the increase that did take place. The increase in the energy gradient is due to the increase in the value of the quantity $\frac{p\,V_s}{V}$ upon which, as may be seen in Eq. 19, the energy gradient is directly dependent.

The plotting of Fig. 11 indicates a value of the coefficient K of very nearly unity in most cases. The solid lines drawn through the groups of points have slopes which, in most cases, are approximately equal to 1. A slope of $S = 1$ corresponds to $K = 1$. The indicated values of f, the coefficient of Darcy's equation for pipe flow, were computed on the basis of the energy gradient for zero solids content as determined by the extension of the straight lines to the line, $\frac{p\,V_s}{V} = 0$.

The fact that the value of K in this situation is approximately unity is of interest in connection with the plotting shown in Fig. 12. In this curve the value of K, computed from the data obtained by Miss Blatch, is plotted as a function of the quantity $\frac{p\,V_s}{V}$. The most notable feature of this plotting is the apparent existence of a maximum value of K at a value of V which gives a minimum of the energy gradient for a given mixture in the pipe. Also, it is apparent that as the velocity decreases $\left(\frac{p\,V_s}{V}\text{ increases}\right)$, the value of K decreases and may well approach 1.

Fig. 13 contains parts of data supplied by Miss Blatch, and parts by Mr. Howard, plotted in the manner used in Fig. 11 for the data obtained at Climax. It is clear that in the ranges of variables covered in these plottings K is not

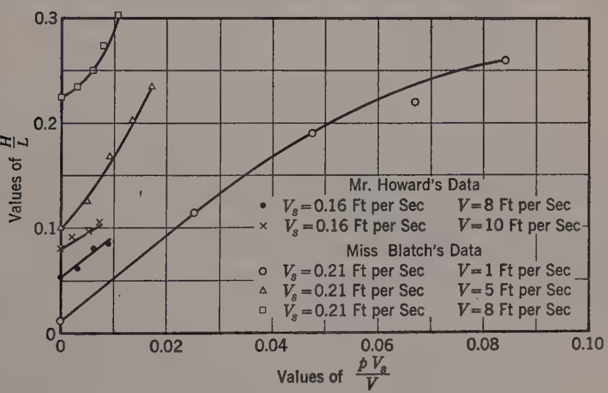

FIG. 13.—ENERGY GRADIENT AS A FUNCTION OF $\frac{p\,V_s}{V}$

unity, whereas it did have a value of practically unity for the data obtained at Climax.

Although adequate data are not available to make a thorough investigation, a superficial examination of the data already cited and some other less complete and reliable data on flow in dredge pipe lines quoted by Miss Blatch[7] indicates that the value of K may decrease as the diameter of the pipe increases.

It is to be expected that the following dimensionless quantities would form a very minimum of parameters with which to describe the coefficient K:

[7] *Transactions*, Am. Soc. C. E., Vol. LVII, December, 1906, pp. 402 and 406.

Reynolds number, **R**; $\frac{V_s}{V}$; the ratio, $\frac{\text{mean diameter of particles}}{\text{diameter of pipe}}$; and percentage of solids (p). Obviously, extensive information is necessary to determine experimentally the laws of the variation of K.

Summary

The fundamental assumption of this study is that the energy, expended in maintaining the flow of a suspension of solids in a liquid, may be expressed as the sum of the energy required to maintain the flow of a homogeneous liquid of the same density, and the energy required to maintain the solids in suspension, through the mechanism of the fluid turbulence overcoming their tendency to settle.

The equations developed on the basis of this assumption lead to conclusions concerning the general features of the flow which are in good accord with the observed facts.

The unique flow observed in the tailings disposal line at Climax is adequately explained in a qualitative manner and certain features of the flow are analyzed quantitatively with good agreement between theory and observation.

In all cases in which complete data are available, and the variation of the coefficient K is not a factor, good agreement is found between theory and observation.

Additional experimental work in which all variables are carefully controlled and observed is essential to obtain a complete understanding of this type of flow.

Acknowledgment

The writer wishes to acknowledge his indebtedness to the officials of the Climax Molybdenum Company for their cooperation in permitting the use of the data, obtained at their plant, in this study. Particular thanks are due to C. J. Abrams, general superintendent, and E. J. Duggan, mill superintendent, for making possible an inspection of the plant and for supplying the detailed information on the flow in the tailings disposal line.

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DISCUSSIONS

PIPE-LINE FLOW OF SOLIDS IN SUSPENSION
A SYMPOSIUM

Discussion

BY ARTHUR L. COLLINS, ASSOC. M. AM. SOC. C. E.

ARTHUR L. COLLINS,[8] ASSOC. M. AM. SOC. C. E.[8a]—A system of measurement has been developed by the writer that is an aid in the solution of some of the important problems encountered in dredge operation. Several measuring units make up the system and are described as follows:

Velocity and Load Meter.—A section of the discharge pipe line can be made to serve as a meter. This requires a horizontal section of discharge pipe, 35 to 50 ft long, to which is attached a chart-recording, differential gage. The drop in pressure due to the resistance to flow generally will be about 20 in. of water pressure, or 0.75 lb per sq in.

As the fluid from the pump passes this section, three stages of flow resistance are recorded: First is the clear-water velocity; and second is the increase in the pressure drop due to the heavier fluid, which may be as great as 50% when the dredge is digging. The third stage may be considered a phenomenon. The pressure drop may increase suddenly as much as 100%, which indicates that there has been some marked change in the distribution of the solids in the fluid.

The records from the so-called load chart obtained from the meter section are supplemented by a gadget that is attached to the end of the pipe line. The object here is to determine how constant the pipe-line velocity is in value. This can be accomplished by the use of a revolving shaft to which are attached steel spokes with vanes on the ends. The vanes dip into the stream flow and provide a displacement record of velocity. The principal purpose of this gage is to obtain relative values. Thus, over a 60-min period the load-meter record, when synchronized with the end of the pipe record, will indicate how the pump responds to the load, in so far as maintaining a continuous velocity is concerned.

In dredge-pump operation it is desirable to use a pump that will maintain a constant pipe-line velocity regardless of the load. If the pump cannot main-

NOTE.—This Symposium appears on pp. 1421–1444 of this issue of *Proceedings*.

[8] Cons. Engr., Berkeley, Calif.

[8a] Received by the Secretary September 24, 1941.

tain a fairly constant velocity, but varies several per cent and has a tendency to slow down under load, it cannot maintain its maximum output continuously.

Tests on small pumps pumping sand under laboratory conditions have led investigators to make broad statements regarding the operating characteristics of a dredge pump. Conventional velocity diagrams, which are purely assumptions, are used to account for variations in pump performance when pumping solids.

Other conditions that have their effect on pump characteristics should be mentioned. The fluid enters the eye of the pump runner at a high velocity and must make a right-angle turn to follow the passages in the pump runner. An additional loss of energy is concealed here that is probably more effective in altering the pump characteristics than the explanation given by the velocity diagram (see Fig. 4).

Slippage in a dredge pump means to the practical dredge operator a spinning of the runner that results in a decreased discharge. The peripheral speed of the pump runner is generally so high that the fluid leaves the runner nearly in a tangential direction. Conditions apparently arise in which the fluid revolves in the case and will not respond to the load pickup. This is another factor that must be taken into consideration in analyzing dredge pumps.

Thus, it must be realized that dredge pumps vary greatly in design. So many conditions enter into the final characteristics of the pump that field tests must be made on each dredge to determine how it can be operated at its best efficiency.

Another gage that is very important to this system of measurement has not been mentioned. This is a depth gage which is attached to the cutter and records on a chart the travel of the cutter. This chart, when synchronized with the load chart, indicates the load that is constantly being picked up by the suction pipe. Each cut or ledge can be rated in terms of load percentage. Thus, the difference in pickup between the port and starboard swings, as well as the shallow and deep cuts, is definitely known.

The following data were obtained from a load chart. It is for the duration of one dredge setup or step:

Cut No.	Time	Load by weight
1	11:30 to 11:37 a.m.	Load is 46.5% starboard swing
2	11:37 to 11:45 a.m.	Load is 21% port swing
3	11:45 to 11:52 a.m.	Load is 25.5% starboard swing
4	11:52 to 12:00 Noon	Load is 9% port swing
5	12:00 to 12:07 p.m.	Load is 23% starboard swing
6	12:07 to 12:17 p.m.	Load is 14% port swing
7–8	12:17 to 12:33 p.m.	Load is in the 3d stage

This overload occurs at the finish of the setup. It is to be noted that the peak pickup amounted to 46.5%, whereas the average is only 23%. Obviously, it should be possible to improve the cutter conditions, the design of the pump, and the method of operating the dredge, and thus increase the output materially.

The chief improvement in dredge operations will not be accomplished by work done in the laboratory on small pumps but by making the large dredge unit a laboratory in itself and recording the various operations.

Tests to determine pipe-flow coefficients fail to take into consideration one important factor that undoubtedly helps to maintain a high percentage of solids in suspension. A large number of discharge lines are made with 15.5-ft sections of pipe with slip joints. The tapered section is 1 in. smaller in diameter than the pipe. Thus in a 24-in. pipe the fluid increases its velocity by 8% at each joint. This has a nozzle effect that diffuses the solids. Even with this restricted area, the pipe coefficient may be as high as 130, using the Hazen and Williams tables.

On the other hand, a smaller pipe (say 15 in.) with long sections and rubber sleeves at the joints may have a coefficient of only 100. Excess friction should react directly upon the dredge output. If by using all the power available it is possible to maintain only 14 ft per sec, it should be possible to increase the output 25% by decreasing the frictional resistance and boosting the velocity to 17.5 ft per sec. In other words, the output will vary as the velocity. There is scarcely a dredge in operation that cannot be improved by applying better engineering technique.

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DISCUSSIONS

AN INVESTIGATION OF STEEL RIGID FRAMES

Discussion

By W. E. Black, Jun. Am. Soc. C. E.

W. E. Black,[23] Jun. Am. Soc. C. E. (by letter).[23a]—Those who submitted discussions have contributed many interesting comments and constructive criticisms. It was gratifying to note that the results of the tests made at the National Bureau of Standards were brought into the discussion frequently. The two investigations represent the only experimental work on steel rigid frames reported in the United States to date (1941), and any discussion of the analysis or design of steel rigid frames should rightly refer to both. Much of the discussion centered about two controversial points—the concentration of stress at the sharp inner corner of the square knee, and the stability of the curved knee. Therefore, each will be discussed individually.

Concentration of Stress in Square Knee.—Several of the discussers have taken exception to the writers' conclusion that the concentration of stress at the inner corner of the square knee was of relatively minor importance. In a continuous and elastic knee of this type, it is generally recognized that the compression stress at the inner corner of the knee is extremely high under the usual manner of loading. Photoelastic tests have demonstrated this fact incontrovertibly. However, it is the writers' contention that, due to the method of fabrication, the type of knee tested does not act as a continuous member. The reactions of the horizontal girder upon the inner face of the column may be assumed to consist of the tension in the top flange, the compression in the bottom flange (both acting approximately at the flange centers of gravity), and a vertical shear. This assumes that the web transmits very little, if any, of the moment or thrust across the connection, a contention that is supported by Fig. 9.

If the action of the knee is represented accurately by the foregoing assumptions, the intensity of stress at the inner corner will be directly dependent upon the type of bearing obtained. In the Lehigh tests, an extreme concentration

Note.—This paper by Inge Lyse, M. Am. Soc. C. E., and W. E. Black, Jun. Am. Soc. C. E., was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1941, by C. J. Posey, Assoc. M. Am. Soc. C. E.; February, 1941, by W. J. Eney, Assoc. M. Am. Soc. C. E.; March, 1941, by Messrs. LaMotte Grover, and William R. Osgood; April, 1941, by Messrs. M. Hirschthal, and Jaroslav J. Polivka; and September, 1941, by A. C. Barrow, Assoc. M. Am. Soc. C. E.

[23] Instr., Dept. of Theoretic l and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

[23a] Received by the Secretary September 3, 1941.

of bearing was encountered, and a high stress resulted. However, the average stress in the compression flange was not excessive. In the Bureau of Standards tests, tight bearing was not obtained, and the maximum stress in the compression flanges at the inner corner of the knee was comparatively low. Mr. Osgood points out that failure in the Bureau of Standards' square knee occurred by local buckling of the compression flanges adjacent to the inner corner. Is it strange that such would be the case when the failure load was two and one half times the "design load," and when any accepted method of analysis will locate the point of maximum stress in the knee at the inner corner? Furthermore, in the usual rigid-frame knee, the compression flange will include cover plates, and local buckling will be less likely to occur, and probably not before the yield point of almost the entire compression flange is reached. However, the possibility of this type of failure should be recognized, and steps should be taken in design to prevent its occurrence.

Mr. Grover warns of the possibility of fatigue failure at the knee. The writer believes that repeated stress will not be an important factor in the design of steel rigid frames of this type since a reversal of stress is not likely to occur. The endurance limit of structural steels, tested from zero to a maximum stress in either tension or compression, is well above the yield point. A fatigue failure at the point of greatest stress concentration, the inner corner, is practically impossible since it is almost always subjected to a compression stress. A tensile failure at the opposite side of the member is equally improbable since the greatest stress there will seldom be as high as the allowable working stress.

One of the worst mishaps that may befall a square type rigid-frame knee is the occurrence of a loose fit at the inner corner. This condition would be equivalent to a decrease in stiffness of the knee. Although complete failure could not result from this cause, there would result excessive stresses and deflection at midspan. On the other hand, an extreme concentration of stress at the inner corner, if not accompanied by local buckling of the compression flanges, would be relieved by the ductility of the metal. In general, even though the stress at the inner corner should reach the yield point, nothing serious would happen until practically the entire compression flange was so stressed, a condition which could not be obtained until the design load was greatly exceeded under the method of design recommended by the writers.

In Mr. Hirschthal's discussion of the maximum compression stress at the reentrant corner of a rectangular rigid-frame knee, he is perhaps underestimating the magnitude of the theoretical maximum compression stress. The determination of this stress is scarcely as simple as Mr. Hirschthal suggests. T. J. Dolan[24] found by photoelastic tests that the maximum compression stress at the reentrant corner of a similarly-shaped bakelite model was about 2.5 times the stress computed by the flexural theory. This stress, however, is extremely local, and in the case of a riveted steel rigid frame, it may be much lower, depending upon the condition of bearing at the corner, the degree of continuity in the knee, and the angle of the reentrant corner.

[24] "An Investigation of Rigid Frames, Part I, Tests of Reinforced Concrete Knee Frames and Bakelite Models," by F. E. Richard, T. J. Dolan, and T. A. Olson, *Bulletin No. 307*, Univ. of Illinois Eng. Experiment Station, November, 1938.

Stability of the Curved Knee.—Attention was focused on the instability of certain types of rigid-frame knees by the results of tests on rigid-frame knees at the Bureau of Standards.[3, 4, 5] A knee with a large circular fillet (very similar to the Lehigh curved knee frame) failed by lateral buckling of the entire curved flange at a load but little greater than the design load. Similar knees having sharp reentrant corners at the inside of the knee supported more than twice their design loads and then failed by local buckling of the compression flange near the reentrant corner. Each of the latter specimens had at least one pair of web stiffeners at the knee. Whether or not web stiffeners would have prevented buckling of the curved knee is problematical. Professor Posey is inclined to think they would, whereas Mr. Osgood expressed the belief that lateral bracing would be necessary to properly stiffen such a knee frame. Mr. Grover suggests that web stiffeners may be used to reduce lateral deflection of the compression flange as a whole, and, in addition, stiffen the outstanding legs of the compression flange against transverse bending of the type found in the Lehigh tests. The writers are not at all certain that web stiffeners would prevent, completely, lateral buckling of the curved flange of a knee of this type. However, the compression flange should be stiffened to prevent transverse bending of the outstanding legs, and a web stiffener welded to or bearing tightly against the compression flange, as suggested by Mr. Grover, might help to solve both problems. Further tests should be made to determine the effectiveness of web stiffeners in preventing lateral buckling of knees of this type.

It is to be regretted that more information regarding buckling was not obtained in the writers' tests, but since the primary purpose of the investigation was the determination of stress distribution, other related phenomena had to be neglected because of time limitations. However, as Mr. Osgood states, with the curved flange supported laterally at one point, no web or flange buckling was noticed in the Lehigh curved-knee frame, even though the flange stresse developed were as high as 29,000 lb per sq in. Further information regarding the ultimate strength of rigid frames may be forthcoming, as the American Institute of Steel Construction has made arrangements for testing to destruction the two rigid frames which were tested under design loads by the writers.

General Discussion.—Professor Eney has described a method of investigation of structural frame models that is particularly well adapted to studying the effects of varying the relative proportions of different parts of the frame. Thus he has investigated the effects of varying the stiffness of both the knee and the footing of a fixed frame, a procedure which could not be followed for larger steel models without prohibitive expense. It is gratifying to note that where his results can be compared with those obtained by the writers, a qualitative agreement was obtained.

The writers are indebted to Mr. Grover for his excellent résumé of the

[3] "Strength of a Riveted Steel Rigid Frame Having Straight Flanges," by Ambrose H. Stang, Martin Greenspan, and William R. Osgood, M. Am. Soc. C. E., *Research Paper No. 1130, Journal of Research*, National Bureau of Standards, Vol. 21, 1938, p. 269.

[4] "Strength of a Riveted Steel Rigid Frame Having a Curved Inner Flange," by Ambrose H. Stang, Martin Greenspan, and William R. Osgood, *Research Paper No. 1161, loc. cit.*, p. 853.

[5] "Strength of a Welded Steel Rigid Frame," by Ambrose H. Stang and Martin Greenspan, *Research Paper No. 1224, loc. cit.*, Vol. 23, 1939, p. 145.

work done in this field. With his recommendations regarding further tests the writers are in whole-hearted agreement.

Mr. Osgood's comment regarding the position of the neutral axis in Fig. 20 is quite correct. The observed neutral axis for stresses on the circular sections would probably fall quite close to the points of zero stress shown in the figure.

Mr. Hirschthal points out the fact that the observed stresses near the tangent points of the curved knee were greater than computed stresses, and that away from the knee, observed stresses were slightly the lower (Fig. 18). It might be well to add that the observed stresses at midspan were slightly greater than corresponding computed stresses. The reason for these differences between computed and observed stresses away from the knee is that the computed stresses were determined from the theoretical external reactions, and there was about 3.5% difference between the theoretical and observed horizontal reactions for the curved knee frame. Using the observed external reactions for the computation of flange stresses away from the knee, the agreement with observed stresses was almost exact.

Mr. Hirschthal's discussion of the curved knee includes one misleading item. If, in Fig. 32, the resultant thrust T is meant to be the resultant of the three forces shown at the section through points 19 and 32, it is somewhat misplaced. Fig. 1(*b*) may be used to clarify the matter. The force T must be equal, collinear, and opposite to the resultant of the pin reactions at the base of the vertical leg (P = 6,000, H = 4,500 approximately), and therefore must intersect the diagonal section of Fig. 32 at a point almost coincident with the center of curvature of the curved flange, and not within the section at all. To resist such a loading, considerable tension, as well as compression, must be developed within the section. A consideration of the forces acting upon a differential rectangle of the web plate at the extreme outside corner of the knee will show that there can be no tensile stress on the diagonal section at that point, although between that point and the neutral axis, considerable tension must exist (Fig. 11).

Mr. Polivka has presented a method of analysis which appears to be particularly well adapted for application to frames of varying moment of inertia, and in which some of the approximations usually made are eliminated. The agreement obtained between Mr. Polivka's computations and the writers' test results was gratifying.

Mr. Barrow has presented a simple and familiar method of computation, which, with a few modifications, gives surprisingly close agreement with the observed test results. The use of such simple methods of analysis is to be encouraged wherever applicable. That Mr. Barrow's computed stresses for the curved flange of the curved knee were somewhat less than the observed stresses demonstrates further that straight-beam formulas are not quite adequate for this type of member.

Even now, more tests on steel rigid frames are being made in the United States, and definite answers may soon be provided for some of the still debatable questions. It would be well to know, among other items, the most economical and efficient shapes for rigid-frame knees, how to stiffen the more flexible types of knees most effectively, and the effect of cyclic or repeated loads on rigid

frames. Nevertheless, the writers are of the opinion that sufficient information concerning the stress distribution in rigid frames and the accuracy of current methods of analysis is now available so that engineers may design rigid frames with confidence and perhaps more economically than has been done in the past.

It is a source of regret to the writer that the preparation of these closing comments could not have been shared by Professor Lyse, now resident in Trondheim, Norway. Distance and disturbed communication conditions, however, have made this virtually impossible.

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DISCUSSIONS

LABORATORY INVESTIGATIONS OF SOILS AT FLUSHING MEADOW PARK

Discussion

BY MESSRS. B. K. HOUGH, JR., AND T. B. RIGHTS

B. K. HOUGH, JR.,[21] ASSOC. M. AM. SOC. C. E. (by letter).[21a]—In his "Synopsis," the author conveys the impression that the tests described in his paper were conducted for certain very practical purposes, such as determination of the general suitability of the Flushing Meadow for the intended development. It is stated (see "Synopsis") that "They [the tests] were made primarily to furnish information on what could be done and how it should be done." The paper itself is disappointing in that the description of the application of test data to practical problems is very meager.

Not only is it a matter of general interest to learn how test data are applied in practice, but, in the writer's opinion, the validity of the data cannot be judged apart from its application. In numerous instances the writer has encountered the popular belief that a given soil has certain characteristic constants which are readily determinable by laboratory tests and which can be applied under any conditions. Opposed to this belief is the theory that testing must simulate (at least in some respect) the anticipated construction procedure, and hence both testing procedure and the type of test data to be secured must be fitted to the proposed work. If space permits in the author's closure, further discussion on application of data might be undertaken.

Other factors bearing directly on the reliability of test results are the equipment and technique used in obtaining samples. Through first-hand knowledge of the Flushing soil formations, obtained during subsurface exploration at various times in Flushing Bay and at the La Guardia Airport, the writer believes that there is some doubt as to the reliability of samples obtained as described by the author, in 3-in. borings. The author's statement that the ratio of maximum stresses in the undisturbed and remolded shear tests was as great as six to one is probably a favorable indication however (see "Physical

NOTE.—This paper by Donald M. Burmister, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1941, by Gordon E. Thomas, Assoc. M. Am. Soc. C. E., and M. N. Sinacori, Jun. Am. Soc. C. E.; June, 1941, by F. J. Kilcawley, Esq.; and September, 1941, by Dimitri P. Krynine, M. Am. Soc. C. E.

[21] Senior Engr., U. S. Engr. Dept., Ithaca, N. Y.

[21a] Received by the Secretary August 29, 1941.

Characteristics"). Another point in connection with sampling is that observations on the number of blows per foot to drive the casing are not as valuable as the number of blows required to drive the sample spoon at each sampling point. The former reflects a highly complex condition of side friction on the casing which is almost certain to increase with depth since the length of casing steadily increases; whereas the latter indicates with some reliability the resistance encountered at each elevation.

The main thought in the paper appears to be that relatively complicated and expensive characteristics tests need not be run on all samples if some correlation can be established between such data and the results of relatively simple routine tests. This thesis is one that merits further development since the objective of soils engineering is to reduce, not to increase, the over-all cost of earthworks construction. For his contribution to this subject, the author is to be commended.

T. B. Rights,[22] Assoc. M. Am. Soc. C. E. (by letter).[22a]—Professor Burmister is to be congratulated on "lifting the veil" on soil mechanics and the mechanics of making tests. However, the average structural engineer (one who has been out of college ten years or more) is still almost as much "in the dark" as he has always been in regard to soil analysis. The writer realizes that this subject is in the transition period. In fact, as far as the writer knows it is only in the last few years that a few books in the English language have been published on this subject. A few years ago engineers were forced to depend upon mimeographed notes or isolated articles in the technical journals. Investigations made by the committee headed by Robert A. Cummings, M. Am. Soc. C. E., laid the groundwork for the researches that have revealed what is known at present.

When soil mechanics started getting publicity about 1935 or 1936, structural men heaved a sigh and said, "Fine, give us the formulas." When no formulas arrived and the structural men discovered that they would have to make expensive and extensive subsurface explorations, they went back to their old method, which is still good if done by an experienced engineer. The old method, according to the writer, consisted in going out and looking at the site, maybe making one loading test of a 1-ft square area, holding a consultation, and stating "Our company has always used a value of 2 tons per sq ft." After spending most of their time on the design of the superstructure, when the foundation settled extensively or tilted, they consoled themselves with the fact that foundations are tricky things and hoped for better luck next time.

More and more subjects are being pushed into the realm of specialization; but structural engineers should have a basic knowledge of the subject of soil mechanics. The design of large structures should be referred to consulting engineers, but the small structures will still have to be built without their services. This is where the average structural engineer should have a basic knowledge of the subject. Where will he obtain this knowledge? Few colleges are giving this subject in extension courses. There is no place for the man

[22] Rosel e, N. J.

[22a] Received by the Secretary September 12, 1941.

residing outside of very large cities to study this subject, except through extensive reading. Many libraries, even in the larger cities, do not have reference material in soil mechanics. It is not surprising that many engineers and designers do not have enough knowledge to design foundations properly. An example of this was recently reported in the metropolitan papers. A building within 5 miles of Flushing Meadow Park is settling badly. Near the beginning of the news article, the engineers are reputed to have stated that the end of the settling process would occur in two weeks. Near the end of the same article was another statement that no one can tell when the settlement will stop.

These news statements should be contrasted with what was done at Flushing Meadow.[23] After the settlement analysis for the Flushing Meadow grounds had been computed, settlement contours were plotted. Additional fill was placed so that, at the end of the 2-yr settling period, the ground would be properly graded. The same investigation could have been made for the aforementioned building as was made for the New York World's Fair. Although this building is on piles, a complete soil investigation would have determined proper pile lengths to support it without extensive settlements.

A paper like that by Professor Burmister does much to bring before the engineering profession a scientific method of making a correct and complete settlement analysis. The undisturbed samples taken in these soil investigations were 3 in. in diameter. The writer understands that in Boston, Mass., 5-in. samples are used; 6-in. diameter samples would produce more accurate results in soil investigations. The possibility of the disturbance in 3-in. samples is very great. If Professor Burmister and others interested in soil mechanics can persuade clients and those paying for undisturbed samples to use 6-in. sizes, they will do much to establish foundation practice on a scientific basis.

The writer believes that the correlation diagrams in Fig. 4 will prove a "milestone" in soil investigations. If another diagram showing voids ratio versus shear strength is added, it will act as a partial check on the other tests. If a sample falls in the same shear-strength range as the other samples, but the compression index falls outside the same range, it might be presumed that the running of the compression test might be in error. If the sample falls outside the shear-strength range, investigation could be made to see whether a real undisturbed sample was used or whether the sample was disturbed or remolded in either the sampling operation or at some later period.

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[23] "The Application of Soil Mechanics in Building the New York World's Fair," by George L. Freeman, George W. Glick, and Hamilton Gray, *Civil Engineering*, October, 1940, p. 649.

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DISCUSSIONS

CONCRETE IN SEA WATER: A REVISED VIEWPOINT NEEDED

Discussion

BY MESSRS. E. C. JACK, AND ALFRED M. FREUDENTHAL

E. C. JACK,[30] ESQ. (by letter).[30a]—Any viewpoint of the subject of concrete in sea water should be somewhat philosophical, in that disintegration or deterioration may or may not be serious, depending upon the rate, and keeping in mind the anticipated life of the structure. So many of the construction people with whom the writer does business consider 10 or 20 years a long time, and anything that might happen after that they believe is immaterial. However, one needs only to look back over the centuries to realize that this matter of ships is a long-range, continuing one extending far into the future. The U. S. Navy is primarily interested in ships, and waterfront improvements to serve the ships should be provided with an eye to a life of at least 100 years. Thus, what to one person might be a negligible breaking up of the structure, to another would be a serious indication when a long life had been planned.

It has been noted that, in almost all cases where disintegration of one kind or another has taken place, there are contributing factors upon which the onus of the situation can be placed, thus clearing sulfate attack of the charge. Nevertheless, it is agreed with Mr. Hadley that the weakness of concrete toward sulfate attack has perhaps been over-stressed. Chemists have their catalytic agents and metallurgists their eutectics; and, in a similar manner, is it not probable that the weakness of concrete toward sulfate and acid attacks makes possible the disintegration by other means, such as wave and frost action, abrasion, and chemical action? In several places in Alaska, notably at Kodiak and at Dutch Harbor, concrete has met with progressive disintegration between tide lines, whereas the remainder of the structure has withstood weathering very well for 25 years. Surely these cases must be a combination of sea-water attack plus frost and wave action.

NOTE.—This paper by Homer M. Hadley, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Thomas E. Stanton, M. Am. Soc. C. E.; April, 1941, by Messrs. W. F. Way, and Glenn S. Paxson; May, 1941, by Messrs. Lester C. Hammond, Ladis H. Csanyi, and G. M. Williams; and June, 1941, by Messrs. Harry E. Squire, and J. W. B. Blackman.

[30] Senior Civil Engr., Puget Sound Navy Yard, Bremerton, Wash.

[30a] Received by the Secretary July 23, 1941.

Cement is a wonderful material that has been improved remarkably through the years, and its comparatively low cost, together with the rather universal distribution of the mills throughout the United States, makes possible the building of many structures that would have been impracticable years ago. It is far from perfect, however, and it may be well to review the most well-known weaknesses of cement and concrete.

Aggregate.—The quality and usefulness of concrete, with present-day cements, demand that aggregate be sound, chemically neutral or basic, and free of oil and other deleterious substances, in addition to being well graded. Thus, in some localities, concrete is very costly because of the distance from which aggregate must be brought. On St. Paul Island, in Bering Sea, it was necessary, one time, to carry gravel by ship from San Francisco, Calif. If cement acted slightly different chemically, then a wider range of aggregate might be used.

Proportioning.—The mixture must be a proper one with sufficient cement, and this item of cement, too, is relative: Witness the amount of cement used per cubic yard today compared with the amount used 50 years ago.

Compounding.—The mixture must be put together in the right order, well worked, with one's weather eye open for numerous technical pitfalls.

Chemically Basic.—Cement and concrete in the green stage are strongly basic, and even when well hardened they are slightly basic. They are thus particularly vulnerable to all substances of an acid nature, such as humous, coffee, industrial wastes, sea water, etc. Just what goes on within the mass during hardening is still very much a mystery. A more thorough and complete chemical combination very probably would improve not only the chemical weakness but also reduce the troubles from erratic hardening, and actually reduce the shrinkage.

Porosity.—Concrete made in the best manner known is quite porous under some conditions, and is thus vulnerable to wave and frost action and penetration of salt water to embedded steel, causing spalling and other such deterioration.

Time of Set.—The time of set is not subject to very good control, which very much limits the use of concrete under some circumstances. In fact, this lack of control can be very embarrassing at times, being either too slow or too fast, and this also nearly prohibits its use for repair work between tides.

Rate of Hardening.—This property naturally varies greatly with different kinds of cement, but in all cases it is absolutely dependent upon the temperature—too much so for comfort in cold climates.

Heat of Crystallization.—This factor has been partly taken care of with the modified cements, but it is still difficult to disperse in spite of concrete being a relatively good conductor.

Workability.—In spite of great care taken to proportion cement and aggregate, the resulting mixtures are almost universally harsh; that is, the cement paste is not as smooth as one would like to have it.

Shrinkage With Hardening.—In practically all cases, concrete shrinks with hardening, thus causing troublesome construction joints.

Volume Change.—This phenomenon occurs, with addition or removal of moisture.

The foregoing résumé includes only the more obvious weaknesses. The items are interrelated in many cases. The first three will probably be present always as they are inherent in the universal use of any material of so cosmopolitan a character as cement. Modified cements and the use of admixtures are the result of trying to obviate some of these deficiencies. If a cement could be manufactured that would combine with all of the water ingredient, it would solve many of these problems.

In conclusion, the analysis of the many combinations of circumstances surrounding any one incident of deterioration is a complicated problem, and it has not been found possible in many cases to put one's finger definitely on any single underlying cause. Generally, it is a combination of poor circumstances, or a poor combination of circumstances, which made the deterioration possible. Therefore, sea-water attack by itself, it is believed, is seldom the cause of progressive disintegration, but in combination with other weaknesses it probably is a contributing factor.

ALFRED M. FREUDENTHAL,[31] ASSOC. M. AM. SOC. C. E. (by letter).[31a]—In attempting to present the problem of resistance of concrete in marine work "in a nutshell," and in stressing the overwhelming importance of resistance against mechanical attack, Mr. Hadley is to be highly commended.

However, there are few engineering problems less suited to the dogmatic approach than that dealt with by the author, and if the question—"Does any form of deterioration occur in the concrete in sea water that would not develop if the exposure were changed to similarly agitated, similarly fluctuating, fresh water?"—is answered definitely in the negative, this appears to be overstating the point.

The deteriorating action of the sea is of two kinds: Mechanical and chemical. The lack of distinct evidence of chemical corrosion in concrete of adequate design and composition (on which evidence the author rests his conclusions) indicates that two "lines of resistance" may be assumed to exist, the "advance line" of the two being the resistance of the structure to mechanical attack. However, there is definite evidence of chemical attack of sea water on the cement and the conclusion that " * * * there appears to be no valid basis for believing that such attack occurs" (see heading "Conclusions") is in contradiction to the results of very extensive and very careful research in America as well as in different countries of Europe.[32]

It is not only the sulfate-of-lime attack that leads to disintegration of concrete; the tricalcium aluminate-hydrate is by far more injurious, although less frequent. Evidence of other corrosive effects of sea water, as well as of the interrelation between the specific cement qualities and the intensity and speed of deterioration, has remained inconclusive, and additional research is needed urgently.

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[31] Res. Engr., Marine Trust, Tel. Aviv Port, Tel. Aviv, Palestine.

[31a] Received by the Secretary July 29, 1941.

[32] "Resistance of Cements to Attack by Sea Water and by Alkali Soils," by Thomas E. Stanton and Lester C. Meder, *Journal*, A. C. I., Vol. 9, 1938, pp. 433–464; "Report of International Congress for Testing Materials," London, 1937 (see especially reports by H. Kuebl, F. Ferrai, and A. Steopoe; and "Deterioration of Structures of Timber, Metal and Concrete Exposed to the Action of Sea Water," Dept. of Scientific and Industrial Research, H. M. Stationery Office, London, 1935 (report by R. E. Stradling).

Questions of serious import are: "Is the effect of puzzolanic admixtures physical (that is, do they increase the density), or is it chemical?" The use of quick-setting cement in marine work creates an early mechanical resistance to external forces. "Do the mechanical advantages outweigh the chemical disadvantages?" The writer has observed conclusive evidence that aluminous cements are very resistant to the action of sulfur compounds in fresh ground water but has further observed that they are definitely weak in sea water. "Why then do they prove definitely weak in marine structure?" "Is this weakness due to physical or chemical action?" "Can pre-cast members be protected against corrosion by applying a bituminous coating to the surface?" Such investigations were begun at Tel-Aviv Port in 1939 in connection with the driving of concrete piles.

The foregoing questions are of considerable importance, and the opinion expressed by the author (see heading "Summary")—that "There is no particular reason for research * * *" is scarcely justified. If the metaphor used by Mr. Hadley were correct, the resistance against chemical corrosion might be considered a comparatively "strong link" in the "chain" of total resistance. This point is supported only by the author's testimony, since the structures referred to do not demonstrate more than the fact that, in all cases where sufficient resistance of the concrete against mechanical attack has been insured, the amount of chemical corrosion has appeared insignificant. The metaphor of the "chain" and its "links" is not a happy one, since the wearing down of both kinds of resistance—the mechanical and the chemical—is not simultaneous but rather consecutive. Considerable deterioration is always preceded and initiated by deficiencies in the mechanical resistance, and there is nothing to support the opinion that after the breakdown of this "first line" of mechanical resistance the subsequent deterioration is not considerably intensified and accelerated by the chemical attack.

Observation of marine structures shows that the main damage to the concrete occurs within the tidal range as well as on horizontal surfaces above sea level (such as flat tops of breakwaters and sea walls) subject to constant spray. It is due to the evaporation and consequent concentration of the attacking saline solution. The fact that the strongest mechanical action of the waves usually occurs near the mean sea level, and thus within the tidal range (wearing down the resistance of the concrete against mechanical attack more rapidly than elsewhere), makes this range particularly susceptible to the concentrated chemical attack beginning in the eroded cavities and voids. Since the corrosion of the permanently submerged parts of marine structures is almost nil, due to the formation of a siliceous gel film that protects the surface, it is not the actual saline concentration of the sea water that is relevant, but the high concentration of the saline solution caused by evaporation. The circumstance that the eroding effect of the initial concentration of the sea water might be regarded theoretically as rather weak thus has no bearing upon the actual intensity of the attack upon concrete marine structures.

It is evident that the density and the impermeability of the concrete, particularly near the surface, are the principal qualities constituting the mechanical resistance of the concrete against attack by the sea. Theoretically,

these qualities may be obtained by adequate grading of the aggregates, rich mixes, and (if this is not thought sufficient) puzzolanic admixtures or the admixture of other dispersing agents. Practically, however, the placing of concrete and its early resistance to mechanical attack is of overwhelming importance in attaining the required density and impermeability, and the best mix will be entirely deprived of its merits if there is even slight neglect in the procedure of placing. The considerable hazards encountered in the construction of marine works are sufficiently known and may not always be forestalled. In spite of all precautions it often may not be feasible to secure practically perfect conditions of placing and to prevent local damage of the concrete surface. Thus its capacity to resist mechanical attack is affected. If no resistance against chemical attack is provided for, relatively rapid deterioration of the structure will start locally, and it will extend gradually over parts initially unaffected. Adequate capacity to resist chemical attack, however, will counteract the injurious effects of local erosion to a considerable degree.

Although the author's statement that "The vitally important matter for concrete used in marine work is to have dense, impermeable concrete made of sound materials with adequate cover over reinforcement" may be fully indorsed, his opinion that "* * * there appears to be no valid basis for believing that such attack occurs" is definitely contradicted by actual evidence (see heading "Conclusions"). The conclusion (see heading "Synopsis") "* * * special precautions against sulfate attack are needless" is not justified, and its application in practice would deprive concrete marine structures of a highly valuable "second line of resistance."

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ANALYSIS OF BUILDING FRAMES WITH SEMI-RIGID CONNECTIONS

Discussion

BY JAROSLAV J. POLIVKA, M. AM. SOC. C. E.

JAROSLAV J. POLIVKA,[17] M. AM. SOC. C. E. (by letter).[17a]—A complete analysis of building frames with semi-rigid connections is presented in this interesting and valuable paper. The authors have corroborated their analytical methods by tests. Both methods applied (slope deflection and moment distribution) have the same disadvantage—namely, that the computations must be made separately for each type of loading, and the exact and refined method, demonstrated by the authors, becomes complex and tedious, even for such a simple case as a two-story building with three equal bays (Fig. 8). The authors agree that the method is not applicable to ordinary design. Three types of loading (Fig. 14) require the determination of sixteen unknown slopes and this number increases to forty eight for an unsymmetrical four-story building.

The writer has attempted to find the answer to three questions relating to the analysis of building frames with semi-rigid connections:

(1) How are the results affected by certain assumptions that would simplify the analysis considerably?

(2) Is it possible to determine certain constants of a given building frame that would be applicable for any type of loading?

(3) What are the short cuts of a general exact method permitting sufficient accuracy in the analysis?

Using the method of elastic weights,[18] two characteristic points for each structural member can be determined. The characteristic points are constants of the building frame and represent sufficient data to analyze the frame for any

NOTE.—This paper by Bruce Johnston, Assoc. M. Am. Soc. C. E., and Edward H. Mount, Esq., was published in March, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1941, by Maurice P. van Buren, Assoc. M. Am. Soc. C. E.; June, 1941, by Wayne W. Smith, Leonard P. Zick, Jr., Juniors, Am. Soc. C. E., and Conrad C. Wan, Esq.; and September, 1941, by Messrs. S. D. Lash, Dean F. Peterson, Jr., and R. W. Stewart.

[17] Research Associate, Univ. of California, and Cons. Engr., Berkeley, Calif.

[17a] Received by the Secretary July 18, 1941.

[18] "Graphical Methods of Analyzing Statically Indeterminate Structures," mimeographed lectures by J. J. Poli vka, Berkeley, Calif., 1940 and 1941.

type of loading, either vertical or horizontal. The characteristic points are centers of simultaneous elastic rotations and can be determined by the following relationships (see Fig. 20):

$$i_{AB} = \frac{L\,G}{3\,(G + 2\,G_{AB})} \quad \text{(26a)}$$

and

$$i_{BA} = \frac{L\,G}{3\,(G + 2\,G_{BA})} \quad \text{(26b)}$$

in which, in addition to the notation of the paper, G is the elastic weight of the beam $\left(= \frac{L}{E\,I} \right)$; and G_A and G_B are the elastic weights of the supports A and B. Angular rotations at A and B due to a unit moment are G_{AB} and G_{BA}.

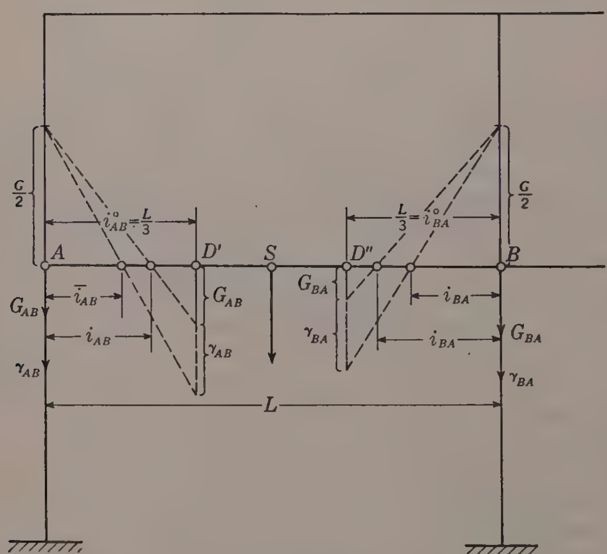

FIG. 20

For semi-rigid connections Eqs. 26 become

$$\ddot{i}_{AB} = \frac{L\,G}{3\,[G + 2\,(G_{AB} + \gamma)]} \quad \text{(27a)}$$

and

$$\ddot{i}_{BA} = \frac{L\,G}{3\,[G + 2\,(G_{BA} + \gamma)]} \quad \text{(27b)}$$

In the simple case in which the width of a member is assumed equal to zero (Fig. 20), the centers of rotation D' and D'' relating to rigidly restrained supports at A and B are distant $\frac{L}{3}$ from the supports.

For a finite member width, this distance can be determined graphically, as shown in Fig. 21, or algebraically by

$$i^{\circ} = \frac{G\,[(l + 2\,b)^2 + 2\,b\,(l + b)] + 12\,\gamma\,b\,(l + b)}{3\,(G + 2\,\gamma)\,(l + 2\,b)} \quad \text{.........} (28)$$

Using the values in the authors' example—$G = 0.04507$, $l = 160$ in., $b = 4$ in., and $\gamma = 0.01775$—the distance $i^{\circ} = 36.223$ in. Since the carry-over factor is

$$r = \frac{i^{\circ}}{L - i^{\circ}} \quad \text{........................} (29)$$

the accuracy of this method may be checked with the results obtained by the authors: $i^{\circ} = \dfrac{36.223}{131.777} = 0.27488$ (compared with 0.275 found by the authors).

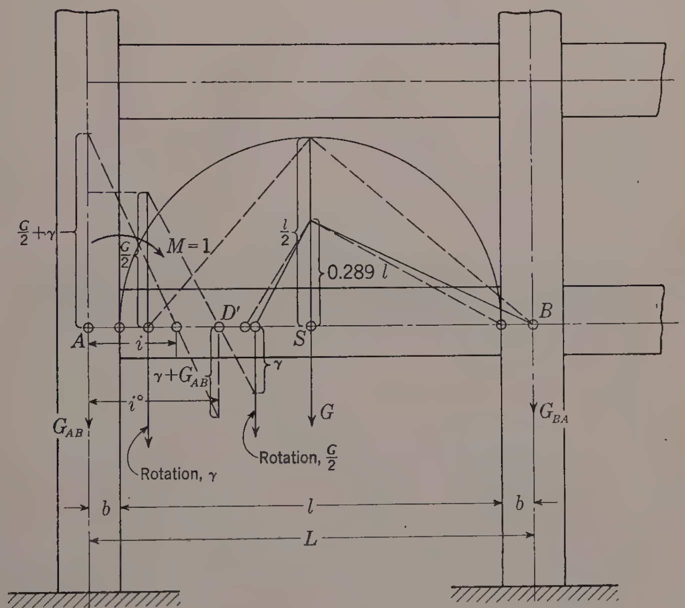

Fig. 21

In the example discussed (Fig. 8), the elastic weight of support of the beam 3-4 results in

$$G_{3-4} = \frac{G_{3-5}\,G_{3-1}}{G_{3-5} + G_{3-1}} \quad \text{.......................} (30)$$

That is, $G_{3\text{-}4} = \dfrac{0.00878 \times 0.01008}{0.00878 \times 0.01008} = 0.00469$ and the center of resultant rotation is determined by $i = i^{\circ}\,\dfrac{G + 2\,\gamma}{G + 2\,(G_{3\text{-}4} + \gamma)} = 32.444$ in. Practically the same value is obtained using the simple term $i = \dfrac{L\,G}{3\,[G + 2\,(G_{3\text{-}4} + \gamma)]}$

$+ b = 28.058 + 4 = 32.058$ in., the difference for this short cut being only 1.2% affecting the center moment of the beam on the side of safety. This simplification eliminates entirely the complex term in Eq. 30.

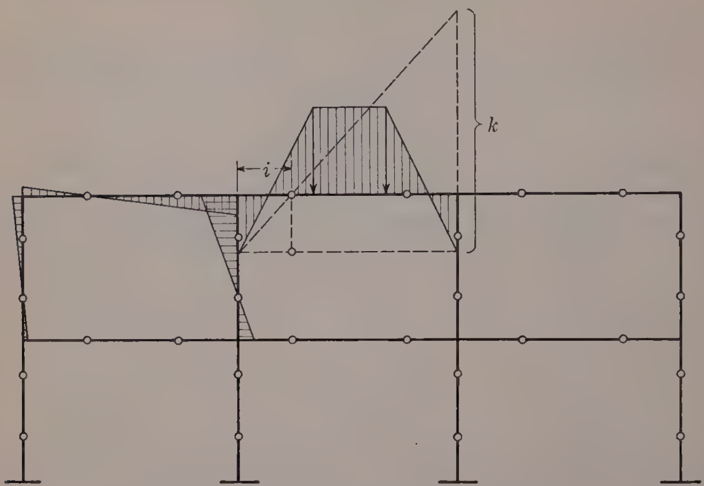

FIG. 22

Knowing the characteristic points of each structural member determining all carry-over factors, the moment diagrams for any type of loading can readily be plotted, as shown in Fig. 22.

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DISCUSSIONS

OPERATION EXPERIENCES, TYGART RESERVOIR

Discussion

BY NICHOLLS W. BOWDEN, M. AM. SOC. C. E.

NICHOLLS W. BOWDEN,[5] M. AM. SOC. C. E.[5a]—The engineering profession in general, and hydraulic engineers in particular, who are interested or engaged in the operation of large reservoirs, or problems connected with them, should feel indebted to the authors for their clear, detailed, and comprehensive treatment of the Tygart Reservoir. There is a dearth of such material both in technical journals and in textbooks. Because of his connection with this and the other proposed storage projects above Pittsburgh while employed by the Corps of Engineers during several years of investigation and planning of these projects and until the construction of the Tygart Dam was well under way, the writer was greatly interested in this timely paper.

The dire need for this reservoir cannot be questioned when one considers the situations brought about by the severe drought in 1930 and the destructive flood in 1936. At times during the drought the natural flow in the Monongahela River was so small as to be negligible—far from enough to provide for lockage of boats and leakage through the locks and dams—and some of the pools in that river above the mouth of the Cheat River became practically empty basins. Below the confluence of the Cheat River, where the bulk of the commerce on this heavy-traffic waterway moves, water released from a privately owned power reservoir on the Cheat River was sufficient in the emergency to supply the pools and maintain navigation. The ravages of the 1936 flood, particularly in the "Golden Triangle" of the City of Pittsburgh, are too well known to need comment here.

As to the efficacy of the Tygart Reservoir in regulating flood flows and affording an adequate, dependable flow during the season of low natural discharge, the authors show quite conclusively that operations during the first two years have been successful. Very properly they state that: The reservoir

NOTE.—This paper by Robert M. Morris, Esq., and Thomas L. Reilly, Esq., was published in April, 1941, *Proceedings*.

[5] Prin. Hydr. Engr., and Head, River Control Section, TVA (Formerly Senior Engr., and Chf., Projects Div., U. S. Engr. Office, Pittsburgh, Pa.).

[5a] Received by the Secretary July 2, 1941.

controls only about 6% of the total drainage area of the Allegheny and Monongahela rivers above Pittsburgh; that the flood storage capacity equals only 4.5 in. of runoff over the contributing area above the dam; and that this capacity is insufficient to store the entire runoff from great storms such as frequently occur in the eastern part of the United States. The relatively small part of the runoff from the drainage area above Pittsburgh that this reservoir regulates is a potent argument for constructing additional reservoirs—and this is now being done—to regulate runoff from a larger part of the area. The relatively small flood-storage capacity of the reservoir seems to merit further discussion, as it is believed that this feature is of the utmost importance in the successful operation of this and other reservoirs. Also, the writer believes that there may be some who do not think that the degree of control which this reservoir affords is adequate.

Various criteria have been advanced for planning and designing reservoirs for flood control in this section of the United States. Some of these may be stated as follows: (1) That the flood-storage capacity should be equivalent to 8 in. or more of runoff over the controlled area; (2) that the flood-storage capacity plus a constant release from the reservoir at bankfull stage downstream, over a given flood period of, say, three or four days, should equal 8 in. or more of runoff; or (3) that the flood-storage capacity should be equivalent to at least one half the average annual runoff. All of these are desirable requirements, if attainable. The capacity of Sacandaga Reservoir in the Hudson River Basin is equivalent to almost 14 in. of runoff; that of the flood-storage space in Norris Reservoir in the Tennessee River Basin, about 12.5 in.; and that of the flood storage space in Lake Mead in the Colorado River Basin, more than three fourths of the average annual runoff. These multiple-purpose reservoirs easily meet the foregoing specifications for flood storage, as no doubt other reservoirs do, and there should be no question as to their suitability for flood control.

Some years ago a prominent engineer remarked to the writer, in a facetious manner, that the planning of large reservoir projects in the Ohio River Basin came about 100 years too late; otherwise, a dam might have been built at or near the mouth of each tributary stream of sufficient height to control flood flows from above and then one on the lower reaches of the Ohio River itself, behind which all of the flood flow from the main-stream drainage could be controlled. This was just a trite way of saying that human occupancy of the valleys with cities, towns, industries, and rail and highway transportation lines had progressed so far that very large reservoirs were no longer feasible. He might have added with equal force that this human occupancy, which had much too often invaded the flood plain, made flood control increasingly important.

Thus, the situation in the Pittsburgh District that confronted engineers who planned the reservoir systems on the Allegheny and Monongahela rivers was one of very great need for flood regulation, as well as for other purposes, and usually of very definite limitations in storage possibilities except at high cost. If the engineers had adhered rigidly to conservative requirements for runoff-

storage relationships, a reservoir plan suitable for the protection of the Pittsburgh area might not have been developed. Instead, the studies proceeded on the basis of securing as high a degree of runoff control as possible, giving due regard to the ratio of estimated annual costs to estimated annual benefits.

In order that the writer may not be misunderstood, it should be stated that he is a firm believer in providing ample storage space in reservoirs for complete control of flood runoff wherever feasible, but he does not believe that this important factor should be allowed to control to the exclusion of all other considerations. As the technique for operating large reservoirs improves—and it is improving rapidly—many concepts of yesterday and even of today will undergo marked change. The importance of collecting adequate precipitation data, and translating them quickly and accurately into stream flow, is now recognized as never before; the technique of routing flood flows in open channels and through reservoirs has advanced to a marked degree in recent years; and last, though probably far from least in possibilities of influencing and perhaps revolutionizing the method of operating reservoirs, is the effort being made by the U. S. Weather Bureau to forecast the location, amount, and intensity of rainfall one or more days in advance. Their initial coöperation in work of this character with the Tennessee Valley Authority in the Tennessee Basin, which was begun in 1939, is being followed closely to see what degree of dependability and accuracy can be attained.

Since the space available in the Tygart Reservoir for storing flood water is insufficient for complete control of the contributory area, it is essential to operate the reservoir in such a manner that the space is used when it will be most effective in accomplishing as much flood-stage reduction as practicable at the critical point downstream, and so that the space again will be made available in the reservoir as promptly as possible for subsequent use. The authors show that this principle has been applied with success in the regulation of floods thus far. Obviously, such operations require very careful attention and the exercise of good judgment in the use of the storage space, lest a storm should exceed expectations and produce a volume of runoff beyond estimates either in the form of a higher or broader inflow hydrograph, or possibly one with a double peak like the flood of January and February, 1939. A full reservoir at the wrong time might conceivably result in a high discharge rate from the reservoir at such time as to synchronize with peak flows at one or more critical points downstream. Involved in this problem is the matter of determining whether to attempt the maximum stage reduction, through the use of all available flood storage space to regulate a current flood, or whether to be satisfied with a smaller stage reduction and thus retain storage space as a factor of safety for use in case subsequent flood flows should develop.

The existence of operating problems of this nature, although difficult of solution, should not of itself deter engineers from constructing large multiple-use reservoirs that do not afford as much storage space for the several purposes as might be desired. The importance of the need and the value of such a project to society should have great influence. The writer has complete con-

fidence in the ability of engineers to develop operating techniques, where necessary (such as the authors indicate is being done in the case of the Tygart Reservoir), which will depend for success on the proper handling of water rather than on storing all flood runoff in a catchment basin so large as to be almost foolproof in operation. Likewise, he believes that engineers are resourceful enough to perfect means for the efficient operation of large multiple-purpose reservoir systems.

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DISCUSSIONS

SURFACE RUNOFF DETERMINATION FROM RAINFALL WITHOUT USING COEFFICIENTS

Discussion

By Messrs. C. E. Ramser, LeRoy K. Sherman, A. J. Schafmayer, C. S. Jarvis, G. W. Musgrave, and F. L. Flynt

C. E. Ramser,[17] M. Am. Soc. C. E.[17a]—The science of hydrology as applied to practical engineering problems has been advanced notably by this paper. The method presented is especially appropriate for use in storm-sewer design where large expenditures are involved. With an increase in rainfall and runoff data made available through hydrologic research, no doubt the method can be simplified greatly. This is true particularly in connection with the estimation of runoff from agricultural areas where, quite often, the cost of improvements would not justify a large expenditure for a detailed analysis of the various hydrologic factors.

LeRoy K. Sherman,[18] M. Am. Soc. C. E.[18a]—A large amount of valuable scientific research and study has been made on the phenomena of infiltration since 1930. The use of the coefficient of runoff has been condemned as irrational and erroneous. Little has been presented in applied hydrology that would induce the engineer to discard the coefficient and make use of the infiltration theory. In this connection the authors' paper is timely.

Some of their remarks on variable conditions—"not yet fully agreed on the mechanics of infiltration," "probably never be possible to reduce it to a simple routine," etc.—may lead the reader to the conclusion that the application of the infiltration theory is still too complex to serve as a working tool. It will be constructive, therefore, to emphasize the major elements of the infiltration theory that are now accepted as facts. In the writer's opinion, they include the following points noted by the authors:

Note.—This paper by W. W. Horner, M. Am. Soc. C. E., and S. W. Jens, Assoc. M. Am. Soc. C. E., was published in April, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1941, by L. L. Harrold, Assoc. M. Am. Soc. C. E.

[17] Chf., Hydrologic Div., SCS, U. S. Dept. of Agriculture, Washington, D. C.

[17a] Received by the Secretary July 21, 1941.

[18] Cons. Engr., Chicago, Ill.

[18a] Received by the Secretary July 28, 1941.

(*a*) "* * * curves [of infiltration capacity] all have characteristically similar shapes, * * * relatively high in the beginning of precipitation, decreasing rapidly * * *, and tending to reach * * * definite minimum values * * *" (see heading "Character of Infiltration-Capacity Curves").

(*b*) "* * * the march of infiltration-capacity rates is closely correlated with mass infiltration" (see heading "Infiltration Capacity"). The writer would supplement this item with the following: The curve of infiltration capacity is readily converted into a curve of mass infiltration. In this form it furnishes a simple device for estimating the volumes of runoff due to real or hypothetical storms.[19]

(*c*) Infiltration capacity varies: (4) With vegetal cover (and hence with season); (2) with soil moisture and soil porosity; and (1) little with surface slope (see heading "Character of Infiltration-Capacity Curves").

(*d*) "Infiltration has ceased to be a vague phenomenon; * * * infiltration expressed in terms of infiltration capacity may now be introduced into engineering practice" (see heading "Relation of Precipitation Rate, Infiltration-Capacity Rate, and Excess Rainfall").

The writer concurs with the foregoing statements of the authors. The scientist here can select many exceptional cases, flaws, variations, and inaccuracies in his search for perfection. The engineer, however, will find that these statements of facts, combined with allowable approximations, can furnish reliable working procedures for estimating infiltration and runoff.

The authors' procedure for correlating the periods of observed rain intensities with the curve of infiltration capacity is a practical requirement in the application of the infiltration theory. Their presentation of this detail appears a little involved. Since infiltration capacity decreases with an increase of soil moisture in the manner shown by the capacity curve, it follows that the equivalent time T, referred to the capacity curve, is equal to the volume of infiltration divided by the average infiltration capacity. Thus, if rain fell for two hours at an intensity of 0.2 in. per hr, and the infiltration capacity was 0.4 at the beginning of rain, then T, due to the first hour of rain, is $\frac{0.2}{0.4} = 0.5$ hr. At the beginning of the second hour of rain, the volume of infiltration had been increased by 0.2 in. and, by the mass curve of infiltration, this lowered the infiltration capacity to 0.35. For the second hour of rain T is $\frac{0.2}{0.35} = 0.57$ hr. The equivalent value of T for the two hours of this rain is 1.07 hr on the capacity curve. Also, if the rain excess (observed surface runoff) for a storm was 1.5 in., and the derived value of average areal infiltration capacity was 0.3 in. per hr, then the equivalent duration time of the storm $T = \frac{1.5}{0.3} = 5$ hr.

In Part II the authors have described in detail the phenomena and procedure of computing surface runoff from small areas. This is predicated upon the use of curves of infiltration capacity. They have given little space in this paper to

[19] "The Unit Hydrograph and Its Application," by L. K. Sherman, *Bulletin*, Associated State Eng. Societies, April, 1941.

the methodology for deriving this curve. They do, however, refer the reader to several papers[8] on the subject.

One of these papers,[20] in the writer's opinion, is a valuable and essential prelude to the authors' paper. The methodology in this article for deriving the curve of infiltration from observed rainfall and runoff applies to small basins in which the downpours of rain are readily correlated with the corresponding volumes or hydrographs of runoff. For small areas, this methodology of derivation is ideal in both simplicity of procedure and accuracy in results. For larger areas, or for small plots on which runoff is measured volumetrically without the hydrograph, the procedure is not directly applicable.

No mention is made, in the paper, of the derivation or use of average areal infiltration capacity. This procedure is in general use. It was first presented by Mr. Horton in 1937.[21] The methodology is simple and applicable to all basins, large or small. On small areas, the initial losses (interception and pocket pondage) may be segregated to give the volume of infiltration and the average areal infiltration capacity (f_a). On large areas, this segregation is generally impracticable. The derived value in this case may be properly termed the "average loss rate" (f_{av}). Studies by the staff of the Illinois State Planning Commission[22] show that reliable capacity or loss-rate curves, and their corresponding mass curves, may be derived from these capacity or loss-rate values.

Attempts have been made by the writer and others to make direct use of these f_a or f_{av} values without subsequent derivation of the curves of infiltration capacity or loss rates. Such attempts, based on averages from unrelated storms, have been futile and misleading. The results are no better than the similar use of a batch of runoff coefficients. Possibly this situation accounts for the authors' silence on the subject of average infiltration capacity.

The authors have made a useful contribution to engineering hydrology. Like all pioneering steps, it can and will be improved and simplified.

A. J. Schafmayer,[23] M. Am. Soc. C. E.[23a]—The method of arriving at the rate of runoff from a given area by deducting the rate of infiltration into the soil plus other losses from the rainfall intensity rate is an interesting departure from former methods. The thorough analysis of the large number of variables

[8] For methodology see:
For Plats—"Analysis of Runoff Plat Experiments With Varying Infiltration Capacity," by R. E. Horton, *Transactions*, Am. Geophysical Union, 1939, p. 693.
"Sprinkled Plat—Infiltration and Runoff Experiments on Arizona Desert Soils," by E. L. Beutner, R. R. Gaebe, Jun. Am. Soc. C. E., and R. E. Horton, *loc. cit.*, 1940, Pt. I, p. 550.
"A Graphical Method of Analysis of Sprinkled-Plot Hydrographs," by A. L. Sharp and H. N. Holtan, *loc. cit.*, p. 558.
For Small Watersheds—"Infiltration-Capacity Values Derived from Small Watershed Data—As Determined from a Study of an 18-Month Record at Edwardsville, Illinois," by W. W. Horner and C. L. Lloyd, *loc. cit.*, p. 522.

[20] "Infiltration-Capacity Values Derived from Small Watershed Data—As Determined from a Study of an 18-Month Record at Edwardsville, Illinois," by W. W. Horner and C. L. Lloyd, *Transactions*, Am. Geophysical Union, 1939, p. 522.

[21] "Determination of Infiltration Capacity for Large Drainage Basins," by R. E. Horton, *loc. cit.*, 1937, Pt. II, pp. 371–385; also, "Determination of Areal Average Infiltration-Capacity from Rainfall and Runoff Data," by R. E. Horton and R. Van Vliet, Office of Land Use Coordination, Flood Control, U. S. Dept. of Agriculture, November, 1940 (mimeographed).

[22] "Application of the Infiltration Theory to Engineering Practice," *Transactions*, Am. Geophysical Union, 1941 (publication pending).

[23] Asst. Chf. Engr. of Sewers, Bureau of Sewers, Dept. of Public Works, City of Chicago, Chicago, Ill.

[23a] Received by the Secretary July 31, 1941.

entering into runoff provides a vivid explanation of the wide variations found in actual runoff for various types of storm. The fact that different agencies, notably the Weather Bureau and Soil Conservation Division of the Department of Agriculture, are gathering more extensive data on rainfall and runoff than heretofore should tend to reduce these errors. Also, the increasing mass of data on stream gaging being secured by the U. S. Geological Survey, in cooperation with various states, should effect further reductions. The present studies by the authors, when compared with these data, will undoubtedly provide further valuable information as to infiltration and the consequent runoff.

The development of the unit hydrograph by LeRoy K. Sherman,[24] M. Am. Soc. C. E., together with the distribution graph by Merrill Bernard,[25] M. Am. Soc. C. E., and the studies in infiltration by the authors and others mentioned in their paper, have added methods by which the designer may make independent checks of his studies of probable runoff.

Also, the authors' careful consideration of the effects on resultant runoff of the varying conditions of surface as to slope, vegetation, method of cultivation, type of soil and its moisture content, and the proportion and disposition of impervious surfaces in any given area, emphasizes the bearing of chance events on any estimate of runoff rates or of the probable frequency of any particular rate.

For the engineer designing storm conduits to carry peak runoffs, a mass of data on simultaneous rain gaging, infiltration, and stream gaging, and simple formulas for its use, should ultimately prove a valuable aid. The material submitted by the authors is a contribution in this direction, since it tends toward removal of one of the unknowns (rate of infiltration) from the coefficient of runoff in the so-called rational formula.

City of Chicago design in recent years has dealt mostly with storm sewers to serve urban areas up to several thousand acres, and the so-called rational formula has been used. The engineer designing sewers or conduits for storm flow is concerned with maximum or peak rates, and, although it is evident that the contribution of the authors may also be of value to engineers concerned with average and minimum rates of flow, the work in Chicago has not been along these lines so that those aspects of the study of runoff are not considered in this discussion.

For peak flows, the commonly accepted formula has been

$$Q = C\, I\, A_w \qquad (13)$$

in which, in addition to the notation of the paper, C is the coefficient, I the intensity rate, A_w the watershed area in acres, and Q the peak runoff. After making determinations required by the method proposed by the authors, the equation could be stated

$$Q = (I - L)\, A_w \qquad (14)$$

in which L would represent the rate of loss through infiltration and pondage. This equation might be considered an intermediate step between the so-called

[24] "Stream Flow from Rainfall by Unit Graph Method," by LeRoy K. Sherman, *Engineering News-Record*, April 7, 1932.

[25] "An Approach to Determinate Stream Flow," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 100, 1935, p. 347.

rational formula and the equation

$$Q = 10{,}000\, P \sqrt{M} \qquad (15)$$

by C. S. Jarvis,[26] M. Am. Soc C. E., or

$$Q = R\, M \qquad (16)$$

as suggested by George W. Pickels,[27] M. Am. Soc. C. E., in which R represents the expected rate of runoff per square mile, M is the area in square miles, and P is a runoff percentage coefficient. With the increasing mass of available data, it may be that, before many years, this last form of equation (Eq. 16) can be used by selecting a runoff factor directly from observed data. This would enable one to avoid both the wide range of probable errors in intensity and frequency involved in applying the rational formula and the extended analytical work involved in the method proposed by the authors. Until such time, however, it will be preferable to use one of the indirect methods.

The application of the infiltration capacity method to a specific problem, as indicated in Part 2, will require considerable adaptation and judgment. The type of city block shown in Fig. 8 and used in the hydrographs in Figs. 5, 6, and 7 differs materially from the normal city block in Chicago. The area, shape, and arrangement of the buildings differ. Also, in Chicago the sewers are usually located in the streets in front of the buildings and the roof drainage connects directly to the sewers. This would advance the crest of the hydrograph somewhat in time. However, since the objective of the designer is to determine the peak storm flow, the time factor, which is important in the rational method, loses its importance in the method proposed by the authors. In Figs. 5, 6, and 7, the outflow hydrographs show a notable uniformity of peak flow for the 60-min, 40-min, and 20-min points whether the storm is of the advanced, intermediate, or delayed pattern. The studies of the flow in gutters for streets and alleys are based upon conditions practically identical with the ordinary conditions in Chicago. The equations and curves (Figs. 12 and 13), therefore, could be applied directly to Chicago calculations under similar conditions without changes in factors.

To apply the authors' method, as they have stated, it is necessary to determine infiltration capacity and overland flow for the particular areas entering into the specific problem. This involves determinations of both initial and continuing infiltration capacities and of the precipitation diagram, and it requires considerable additional investigation for many tributary areas with extensive and tedious computations. Therefore, its immediate application for local studies will probably consist of calculations for runoff, using the rational formula with a coefficient of runoff, and checking and comparing the results with results obtained by other methods, including that suggested by the authors, with estimated values of infiltration capacity, overland flow, and for the precipitation diagram adapted from the data and diagrams submitted by the authors.

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[26] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 364.

[27] "Run-Off Investigations in Central Illinois," by George W. Pickels, *Bulletin No. 232*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., September, 1931, p. 130.

C. S. Jarvis,[28] M. Am. Soc. C. E.[28a]—No matter how commendable the author's purpose may be in attempting to determine surface runoff from rainfall without using coefficients, both the procedure and the objective obtained are subject to challenge—in popular parlance it might be said that the remedy is at least equal to the ills it is intended to cure. Specifically, the obvious implication is that ample data from properly distributed stations are available wherewith to account for the changing intensities and migrations of turbulent centers during the storm period. The painstaking work done in the Soil Conservation Service has demonstrated the wide digressions from point observations ordinarily experienced between stations much more closely spaced than has yet been found practicable on extensive drainage basins. With such spacings of rainfall stations as have been provided over the United States at large, or are now in process, it appears that the authors, in the matter of detail and apparent refinement, may go far beyond the limits of accuracy and reasonable interpretation of basic data, even with such detailed records as they had assembled for a metropolitan center that is especially favored in both length and quality of hydrologic records. It is conceivable that the assumptions involved in the application of this method to less-favored extensive areas by less skilled technicians may include greater errors and uncertainties than some of the more familiar devices.

It is readily conceded that a large army of explorers examining each unit of area in a strange country may ultimately acquire greater volumes and details of information than would a reconnaissance or scouting party; yet comprehensive reconnaissance and rapid scouting with bird's eye views from points of vantage may more nearly satisfy the immediate requirements, and at the same time determine whether the detailed study should be undertaken. Candidly, the writer has deplored the general neglect of voluminous basic data which have been shunted aside during recent years in favor of the microscopic approach, the most satisfactory test and confirmation of which would be the derivation of those same basic data by integration of discharges from elementary areas. Results from individual efforts as well as those of small groups in research projects have brought such returns from a few months' effort as could not be encompassed in years or decades by such procedures as would ban the normal use of coefficients, formulas, and other means of summarizing available observations and experience.

The data in Table 7, representative samples of extensive tabulations ready for publication, illustrate the advantageous use of coefficients, in this instance expressing percentages of the 5-yr (1934–1938) mean discharges of the Mississippi River and some of its principal tributaries, as well as of some minor tributaries with notable records. The discharges for individual years of the 5-yr period, together with the recorded or estimated means for each period progressively increasing by 5-yr steps up to 30 yr, by 10-yr steps up to 60 yr, thence by 20-yr steps up to 80, 100, or 120 yr according to available data, have provided some fundamentals which could not conceivably be attained except by use of coefficients showing relationships that may be used to bridge gaps in

[28] Hydr. Engr., SCS, Dept. of Agriculture, Washington, D. C.

[28a] Received by the Secretary August 8, 1941.

the record. Would it be venturing too far to suggest that the consistent trends and variations from the latest 5-yr means reflected in the percentage coefficients for yearly or longer-period means should provide some guidance for such detailed analyses as the authors have so ably undertaken? Instead of building up exclusively from small increments, using the synthetic method, they would be employing derivations from larger units by analysis of basic data. It would be comparable to making use of the broad perspective views from reconnaissance scouting in order to locate and orient the small plot of strategic importance, and would in no way restrict the detailed studies within the particular plot.

Granting that there is a fundamental difference between the authors' use of instantaneous hydrologic phenomena and the writer's approach through yearly or longer period means, it should be realized that the latter represents the total natural integration of the former, and is fully as susceptible to forecasting, if not more dependable than are the instantaneous or short-period phenomena.

The assertions, such as the authors', that empirical formulas and devices are unscientific and outmoded, including the use of coefficients, do not seem to eliminate them from practice nor from the preferred equipment where basic data are either meager or broken. Surely the results of procedures outlined in this paper may be expressed in the form of coefficients showing what portion of the rainfall appeared as runoff. Whether this percentage is identical with that of excess rainfall or divergent from it, such expression should not be ruled out of reports dealing with hydrologic data. For example, the Ohio River flood volume of March, 1936, at Pittsburgh, Pa., represented about 95% of the rainfall and snow mantle on the tributary watersheds, and about 40% of the total flood discharge occurred on the day of maximum flood. Likewise the Ohio River flood volume of January, 1937, at Metropolis, Ill., represented 69% of the storm precipitation plus the water content of snow mantle, of which not more than 4% was discharged on the day of maximum flood.

Fortunately, the 11-yr record period from 1928 to 1938, inclusive, for Spur, Tex., afforded opportunities for practical tests of runoff determinations by coefficients and by other means. The total precipitation of twenty-eight recorded, runoff-producing storms for this period amounted to 40.89 in., and the observed runoff was 16.26 in. or 40% for watershed No. 1, with 53, 38, 57, and 21%, respectively, for watersheds Nos. 2, 3, 5, and 6. Applying these percentages, as runoff coefficients for the respective watersheds for the entire list of twenty-eight storms that produced runoff within the record period, provides a total of $5 \times 28 = 140$ separate determinations. Comparison of the estimated runoff as derived from those average coefficients with observed runoff resulted in some 41.5% of the number falling within a $\pm$ 20% tolerance zone. Variations of the average runoff coefficient by some arbitrary factors designed to take account of the antecedent rainfall, whether copious, moderate, or meager, would increase the items within the 20% tolerance to nearly 70%, or a result comparable to that achieved with the most rigorous, painstaking, and time-consuming analyses of which the writer has knowledge. However, in view of the time, labor, and specialized technical guidance required for the rigorous and complicated analyses, and because of their susceptibility to gross

TABLE 7.—Record of Mean Discharges at Stream-Gaging Stations of Method," or Estimated from all Available Data, for Various September 30, 1938; the Latest

Item No.	Stream and station	Drainage area, in sq miles	Years of Record: Observed: Gagings	Years of Record: Observed: Gage heights	Years of Record: Gaged and estimated	1937–1938: cfs[a]	1937–1938: Percentage of 5-yr mean
1	Mississippi River, St. Paul, Minn.	36,800	46	61	100	8,218	167.4
2	Mississippi River, LaCrosse, Wis.	62,800	9	64	100	30,390	143.7
3	Mississippi River, Le Claire, Iowa	88,600	65	65	100	49,770	137.5
4	Mississippi River, Davenport, Iowa	88,800		79			
5	Mississippi River, Keokuk, Iowa	119,000	60	71	100	65,520	135.4
6	Mississippi River, Grafton, Ill.	170,000	39[d]	59	100	(99,000)	123.8
7	Mississippi River, Alton, Ill.	171,500	11	49			
8	Illinois River, Peoria, Ill.	13,480	28	67	80	17,410	114.2
9	Illinois River, Beardstown, Ill.	(22,500)	18	60	80	22,260	112.0
10	Missouri River, Fort Benton, Mont.	24,600	57	57	100	6,010	130.4
11	Missouri River, Fort Benton, Mont. (including consumptive use)	24,600	57	57	100		
12	Missouri River, Boonville, Mo.	505,700	13	65	100	40,190	103.4
13	Missouri River, Hermann, Mo.	528,200	39[d]	65	100	56,640	106.3
14	Mississippi River, St. Louis, Mo.	701,000	10	81	100	156,800	116.6
15	Mississippi River, Cape Girardeau, Mo.	716,000	5	47	100	(173,000)	118.9
16	Greenbrier River, Alderson, W. Va.	1,357	41	41	100	2,061	94.8
17	Kanawha River, Kanawha Falls, W. Va.	8,367	61	61	100	13,170	105.4
18	Wabash River, Mt. Carmel, Ill.	28,600	11	54	100	35,020	146.5
19	Cumberland River, Nashville, Tenn.	12,860	50	65	100	(18,000)	90.8
20	Tennessee River, Chattanooga, Tenn.	21,400	64	65	100	35,720	103.3
21	Tennessee River, Florence, Ala.	30,800	44	67	100	49,600	100.8
22	Tennessee River, Johnsonville, Tenn.	38,520	49	62	100	58,330	98.4
23	Ohio River, Pittsburgh, Pa.	19,100		67	100	(32,600)	106.5
24	Ohio River, Sewickley, Pa.	19,500	5	5	100	33,120	106.5
25	Ohio River, Huntington, W. Va.	55,200	4	25	100	85,240	109.7
26	Ohio River, Cincinnati, Ohio	76,580		68	100	(108,715)	107.8
27	Ohio River, Louisville, Ky.	91,200	4	67	100	124,000	106.1
28	Ohio River, Metropolis, Ill.	203,000	12	65	100	269,800	105.4
29	Ohio River, Cairo, Ill.	203,900	39[d]	80	100	(270,700)	105.4
30	White River, De Valls Bluff, Ark.	23,800	11	11	100	30,140	131.4
31	White River, Clarendon, Ark.	25,750	11	54	100	(37,000)	129.0
32	Arkansas River, Little Rock, Ark.	157,900	39[d]	67	120	46,590	129.1
33	Yazoo River, Greenwood City, Miss.	7,450	11	35	120	(11,240)	114.5
34	Yazoo River, Yazoo City, Miss.	(8,500)		54	120	(12,600)	114.5
35	Red River, Garland, Ark.	51,500	9	9	120	27,380	150.8
36	Red River, Alexandria, La.	65,850	39[d]	67	120	(50,000)	166.3
37	Mississippi River, Memphis, Tenn.	932,800	9	67	120	445,500	107.7
38	Mississippi River, Arkansas City, Ark.	1,130,700	39[d]	59	120	550,700	110.1
39	Mississippi River, Vicksburg, Miss.	1,144,500	56	68	120[e]	567,300	109.4
40	Mississippi River, Vicksburg, Miss., by "sampling method"		56	68	120	((583,814))[f]	112.7
41	Mississippi River, Natchez, Miss.	1,149,400	35	105	122[e]	(573,200)	109.5
42	Miss.-Atchafalaya, Red River Landing and Simmesport, La.	1,242,700	65[d]	65	120	(668,000)	114.8
42a	Added items from Mississippi commission reports						

Test of Official Measurements

Item No.	Stream and station	Drainage area, in sq miles	Gagings	Gage heights	Gaged and estimated	cfs	Percentage of 5-yr mean
43	Totals of 13 tributaries, Meramec River, Mo., to White River, Ark., inclusive	43,488	9–17	17	120	50,570	125.8
44	Ungaged intervening areas	27,212			120	(30,700)	108.9
6	Mississippi River, Grafton, Ill.	170,000	39[d]	59	120	(99,000)	123.8
13	Missouri River, Hermann, Mo.	528,200	39[d]	65	120	56,640	106.3
29	Ohio River, Cairo, Ill.	203,900	39[d]	80	120	(270,700)	105.4
32	Arkansas River, Little Rock, Ark.	157,900	39[d]	67	120	46,590	129.1
45	Totals, items 6, 13, 29, 32, 43, and 44	1,130,700				(554,200)	112.0
46	Departures of derived data					+3,500	
38	Mississippi River, Arkansas City, Ark.	1,130,700	39[d]	59	120	550,700	110.1
47	Departures in percentage of the latest 5-yr mean					+0.7	

THE MISSISSIPPI RIVER SYSTEM, EITHER OBSERVED, DERIVED BY THE "SAMPLING PERIODS OF RECORD TERMINATING WITH THE WATER-YEAR, 5-YR MEAN BEING TAKEN AS 100%

Item No.	1936–1937		1935–1936		1934–1935		1933–1934		5-Yr Mean	
									100%	
	cfs[a]	Percentage of 5-yr mean	cfs	Percentage of 5-yr mean	cfs	Percentage of 5-yr mean	cfs	Percentage of 5-yr mean	cfs	csm[b]
1	5,379	109.6	5,330	108.6	3,688	75.1	1,935	39.4	4,910	0.133
2	17,580	83.1	23,570	111.4	22,930	108.4	11,300	53.4	21,154	0.337
3	33,160	91.6	37,390	103.3	41,820	115.5	18,870	52.1	36,200	0.409
4										
5	49,900	103.1	47,680	98.6	57,260	118.4	21,540	44.5	48,380	0.407
6	86,000	107.5	72,200	90.3	106,000	132.5	36,800	46.0	80,000	0.471
7									80,600	0.470
8	16,120	105.7	13,320	87.3	18,590	121.9	10,810	70.9	15,250	1.131
9	21,710	109.2	16,540	83.2	26,540	133.5	12,360	62.2	19,882	0.884
10	3,618	78.5	4,559	98.9	4,307	93.5	4,952	107.5	4,608	0.187
11									(6,000)	0.244
12	38,900	100.1	33,860	87.2	53,560	137.9	27,740	71.4	38,850	0.077
13	59,000	110.7	41,090	77.1	80,010	150.1	29,750	55.8	53,300	0.101
14	146,800	109.1	113,700	84.5	187,600	139.5	67,700	50.3	134,520	0.192
15	162,000	111.4	117,000	80.4	203,600	140.0	71,730	49.3	145,466	0.203
16	2,078	95.6	2,435	112.0	3,007	138.3	1,291	59.4	2,174	1.602
17	11,480	91.8	13,840	110.7	16,180	129.5	7,824	62.6	12,499	1.494
18	39,250	164.2	14,510	60.7	22,710	95.0	8,043	33.6	23,907	0.836
19	(24,600)	124.1	(16,300)	82.2	(23,500)	118.6	(16,700)	84.3	(19,820)	1.541
20	36,810	106.5	38,600	111.7	35,700	103.3	(26,000)	75.2	34,566	1.615
21	53,570	108.9	55,740	113.3	49,650	100.9	37,400	76.0	49,192	1.597
22	67,700	114.2	61,740	104.1	62,540	105.5	46,140	77.8	59,290	1.539
23	(38,900)	127.1	(31,500)	102.9	(29,400)	96.1	(20,600)	67.3	(30,600)	1.601
24	39,520	127.1	32,030	103.0	29,840	96.0	20,960	67.4	31,094	1.595
25	95,110	122.4	79,370	102.1	82,920	106.6	(46,030)	59.2	77,734	1.408
26	(125,971)	124.9	(100,609)	99.7	(111,897)	110.9	(59,009)	58.5	(100,863)	1.317
27	149,000	127.4	111,900	95.7	(131,586)	112.5	(68,195)	58.3	116,919	1.282
28	337,100	131.7	226,600	88.5	285,200	111.4	(161,000)	62.9	255,940	1.261
29	(338,500)	131.7	(227,500)	88.5	(286,400)	111.5	(161,600)	62.9	(256,940)	1.260
30	26,600	116.0	10,110	44.1	32,190	140.4	15,630	68.2	22,934	0.964
31	(35,163)	122.6	10,246	35.7	43,252	150.8	17,709	61.8	(28,674)	1.112
32	35,700	98.9	14,010	38.8	65,400	181.2	18,740	51.9	36,088	0.229
33	(14,900)	151.8	(7,000)	71.3	10,230	104.2	5,720	58.3	(9,818)	1.318
34	(16,700)	151.8	(7,900)	71.8	(11,400)	103.6	(6,400)	58.2	(11,000)	1.294
35	16,726	92.1	7,298	40.2	28,350	156.2	(11,000)	60.6	18,151	0.352
36	(27,000)	89.8	10,885	36.2	42,447	141.2	(20,085)	66.8	30,060	0.456
37	516,700	124.9	349,900	84.6	511,400	123.7	244,285	59.1	413,551	0.444
38	623,600	124.7	387,900	77.6	651,500	130.3	286,900	57.4	500,140	0.442
39	641,200	123.7	401,600	77.5	676,800	130.6	305,000	58.8	518,380	0.453
40	((632,589))	122.1	((405,886))	78.3	((660,815))	127.5	((318,929))	61.6	((518,100))	0.453
41	(647,800)	123.7	(405,740)	77.5	(683,660)	130.6	(308,150)	58.8	(523,667)	0.456
42	(701,080)	120.5	(424,730)	73.0	(772,870)	132.8	(342,665)	58.9	(581,870)	0.468
42*a*			421,610	72.5	750,235	128.9	545,545	59.4		

AND ESTIMATED DISCHARGES

43	49,288	122.6	18,325	45.6	58,247	144.8	25,112	62.4	40,214	0.925
44	(40,000)	141.9	(14,000)	49.7	(44,000)	156.1	(15,000)	53.2	(28,196)	1.036
6	86,000	107.5	72,200	90.3	106,000	132.5	36,800	46.0	80,000	0.471
13	59,000	110.7	41,090	77.1	80,010	150.1	29,750	55.8	53,300	0.101
29	(340,500)	131.7	(227,500)	88.5	(288,400)	111.5	(161,600)	62.9	(256,940)	1.260
32	35,700	98.9	14,010	38.8	65,400	181.2	18,740	51.9	36,088	0.229
45	(610,488)	123.4	(387,125)	78.3	(642,057)	129.8	(287,002)	58.0	(494,738)	0.438
46	−13,112		−775		−9,443		+102		−5,402	
38	623,600	124.7	387,900	77.6	651,500	130.3	286,900	57.4	500,140	0.442
47	−2.6		−0.16		−1.9		+0.02		−1.1	

TABLE 7.—

Item No.	10-Yr Mean		15-Yr Mean		20-Yr Mean		25-Yr Mean		30-Yr Mean		40-Yr Mean	
	cfs	%	cfs	%	cfs	%	cfs	%	cfs	%	cfs	%
1	4,906	99.9	5,346	108.9	6,492	132.2	7,771	158.3	7,723	157.3	8,810	179.4
2	19,947	94.3	(21,900)	103.5	(23,100)	109.7	(24,250)	114.6	(24,100)	113.9	(27,500)	130.0
3	35,845	99.0	37,734	104.2	39,566	109.3	41,456	114.5	41,250	114.0	45,000	124.3
4												
5	48,218	99.7	50,352	104.1	51,880	107.2	54,340	112.3	54,586	112.8	58,320	120.5
6	84,844	106.1	93,130	116.4	93,900	117.4	96,700	120.9	96,700	120.9	101,240	126.5
7												
8	16,405	107.6	17,617	115.5	17,158	112.5	16,700	109.5	16,532	108.4	(17,173)	112.6
9	21,581	108.5	24,354	122.5	23,490	118.1	(22,500)	113.2	(22,500)	113.2	(23,000)	115.7
10	5,122	111.2	6,232	135.2	6,496	141.0	7,181	155.8	7,615	165.3	8,005	173.7
11	(6,500)	108.3	(7,300)	121.7	(7,400)	123.3	(8,000)	133.3	(8,300)	138.3	(8,550)	142.5
12	49,775	128.1	52,955	136.3	(55,000)	141.6	(58,000)	149.3	(58,500)	150.6	(60,000)	154.4
13	59,520	111.7	71,257	133.7	73,165	137.3	75,664	142.0	74,027	138.9	74,900	140.5
14	145,670	108.3	(165,500)	123.0	(168,200)	125.0	(173,500)	129.0	(171,900)	127.8	(177,400)	131.9
15	(154,500)	106.2	(178,400)	122.6	(181,400)	124.7	(187,000)	128.6	(185,200)	127.3	(190,800)	131.2
16	2,038	93.7	2,054	94.5	2,067	95.1	2,086	96.0	2,080	95.7	2,111	97.1
17	11,561	92.5	11,656	93.3	11,821	94.6	11,910	95.3	11,884	95.1	12,510	100.1
18	25,929	108.5	(27,000)	112.9	(27,000)	112.9	(26,800)	112.1	(26,800)	112.1	(26,000)	108.8
19	(20,128)	101.6	20,579	103.8	21,584	108.9	21,483	108.4	21,559	108.8	20,992	105.9
20	35,283	102.1	34,762	100.6	36,702	106.2	36,357	105.2	36,604	105.9	37,131	107.4
21	51,506	104.7	50,604	102.9	53,443	108.6	52,534	106.8	52,960	107.7	52,760	107.3
22	63,205	106.6	62,597	105.6	66,258	111.8	64,538	108.9	65,000	109.6	64,030	108.0
23	(29,500)	96.4	(31,100)	101.6	(31,000)	101.3	(31,100)	101.6	(31,000)	101.3	(30,700)	100.3
24	(30,000)	96.5	(31,600)	101.6	(31,500)	101.3	(31,600)	101.6	(31,500)	101.3	(31,200)	100.3
25	(71,680)	92.2	(73,000)	93.9	(74,000)	95.2	(75,244)	96.8	(75,244)	96.8	(77,600)	99.8
26	(94,696)	93.9	(96,000)	95.2	(98,000)	97.2	(99,814)	99.0	(99,700)	98.8	(102,000)	101.1
27	(111,219)	95.1	(114,000)	97.5	(116,000)	99.2	(117,537)	100.5	(116,000)	99.2	(118,700)	101.5
28	253,970	99.2	(266,500)	104.1	(274,300)	107.2	(273,000)	106.7	(275,500)	107.6	(270,000)	105.5
29	255,060	99.3	267,600	104.1	275,300	107.1	273,900	106.6	276,600	107.6	271,500	105.7
30	24,200	105.5	27,000	117.7	(25,500)	111.2	(27,000)	117.7	(26,300)	114.7	(26,600)	116.0
31	30,250	105.5	(32,000)	111.6	(30,000)	104.6	(30,500)	106.4	(30,000)	104.6	(30,000)	104.6
32	36,200	100.3	40,260	111.6	41,240	114.3	40,000	110.8	37,200	103.1	37,600	104.2
33	10,293	104.8	(10,500)	106.9	(9,500)	96.8	(9,000)	91.7	(8,200)	83.5	(8,800)	89.6
34	(11,500)	104.5	(11,800)	107.3	(10,700)	97.3	(10,100)	91.8	(9,200)	83.6	(9,900)	90.0
35	16,920	93.2	(17,060)	94.0	(17,420)	96.0	(17,060)	94.0	(16,340)	90.0	17,060	94.0
36	30,120	100.2	29,090	96.8	30,630	101.9	29,150	97.0	27,525	91.6	28,870	96.0
37	426,000	103.0	(460,000)	111.2	(472,000)	114.1	(475,000)	114.9	(477,000)	115.3	(480,000)	116.1
38	514,400	102.9	555,800	111.1	574,870	114.9	582,900	116.5	586,900	117.3	588,000	117.6
39	534,240	103.1	(561,500)	108.3	(573,400)	110.6	(574,500)	110.8	(573,800)	110.7	576,300	111.2
40	((533,033))	102.9	((567,500))	109.5	((580,000))	111.1	((588,975))	113.7	((592,000))	114.3	((593,000))	114.5
41	(539,580)	103.0	(567,000)	108.3	(579,000)	110.6	(580,000)	110.8	(579,500)	110.7	(582,000)	111.1
42	596,085	102.4	629,200	108.1	649,800	111.7	644,200	110.7	645,400	110.9	646,300	111.0
											Test of Official Measurements	
43	42,010	104.5	46,407	115.4	(54,400)	135.3	(58,000)	144.2	(57,200)	142.2	(58,200)	144.7
44	(28,526)	101.7	(31,000)	109.9	(34,000)	120.6	(38,000)	134.8	(39,000)	138.3	(36,000)	127.7
6	84,844	106.1	93,130	116.4	93,900	117.4	96,700	120.9	96,700	120.9	101,240	126.6
13	59,520	111.7	71,257	133.7	73,165	137.3	75,664	142.0	74,027	138.9	74,900	140.5
29	(255,060)	99.3	(267,600)	104.1	(275,300)	107.1	273,900	106.6	276,600	107.7	271,500	105.7
32	36,200	100.3	40,260	111.6	41,240	114.3	40,000	110.8	37,200	103.1	37,600	104.2
45	(506,160)	102.3	(549,654)	111.1	(572,005)	115.6	(582,264)	117.7	(580,727)	117.4	(579,440)	117.1
46	−8,240		−6,146		−2,865		−636		−6,173		−8,560	
38	514,400	102.9	555,800	111.1	574,870	114.9	582,900	116.5	586,900	117.3	588,000	117.6
47	−1.6		−1.2		−0.6		−0.13		−1.2		−1.7	

[a] Cubic feet per second.
[b] Cubic feet per second per square mile.
[c] Unofficial, derived, or estimated quantities are enclosed in parentheses; thus: (8,901).
[d] Based on discharge data for the years 1900 to 1931 in Tables 2 and 13, House Document 259, 74th Congress, 1st Session, "Comprehensive Report on Reservoirs in Mississippi River Basin"; together with later official yearly records.

Continued

Item No.	50-Yr Mean		60-Yr Mean		80-Yr Mean		100-Yr Mean		120-Yr Mean	
	cfs	%	cfs	%	cfs	%	cfs	%	cfs	%
1	(8,901)[e]	181.3	(9,330)	190.0	(9,000)	183.3	(8,700)	177.2	....	....
2	(27,300)	129.1	(29,000)	137.1	(29,400)	139.0	(29,000)	137.1	....	....
3	44,690	123.5	47,350	130.8	(48,000)	132.6	(47,000)	129.8	....	....
4										
5	57,120	118.1	60,770	125.6	(62,000)	128.2	(61,000)	126.1	....	....
6	(99,000)	122.6	(104,000)	130.0	(106,000)	132.5	(104,000)	130.0	....	....
7										
8	(16,900)	110.8	(17,400)	114.1	(18,000)	118.0	....	....	....	....
9	(22,700)	114.2	(23,300)	117.2	(24,000)	120.7	....	....	....	....
10	8,281	179.7	(8,413)	182.6	(8,100)	175.8	(7,800)	169.3	....	....
11	(8,740)	145.7	(8,820)	147.0	(8,410)	141.7	(8,050)	138.2	....	....
12	(60,000)	154.4	(64,000)	164.7	(61,000)	157.0	(60,000)	154.4	....	....
13	(72,000)	135.1	(74,000)	138.8	(70,000)	131.3	(68,000)	127.6	....	....
14	(172,200)	128.0	(179,250)	133.3	(176,250)	131.0	(173,200)	128.8	....	....
15	(185,000)	127.2	(192,600)	132.4	(189,000)	129.9	(185,800)	127.7	....	....
16	(2,150)	98.9	(2,250)	103.5	(2,250)	103.5	(2,200)	101.2	....	....
17	12,960	103.7	13,020	104.2	(12,900)	103.2	(12,800)	102.4	....	....
18	(26,500)	110.8	(27,000)	112.9	(27,000)	112.9	(28,000)	117.1	....	....
19	20,806	105.0	(20,725)	104.6	(20,600)	103.9	(21,000)	106.0	....	....
20	37,307	107.9	38,100	110.2	(38,000)	109.9	(39,000)	112.8	....	....
21	(53,000)	107.7	(54,000)	109.8	(54,000)	109.8	(56,000)	113.8	....	....
22	63,620	107.3	(65,000)	109.6	(65,000)	109.6	(67,000)	113.0	....	....
23	(30,800)	100.7	(31,100)	101.6	(30,600)	100.0	(30,100)	98.4	....	....
24	(31,300)	100.7	(31,600)	101.6	(31,100)	100.0	(30,600)	98.4	....	....
25	(80,500)	103.6	(81,000)	104.2	(80,000)	102.9	(79,000)	101.6	....	....
26	(105,000)	104.1	(105,800)	104.9	(104,000)	103.1	(103,000)	102.1	....	....
27	(122,000)	104.3	(123,000)	105.2	(121,000)	103.5	(120,000)	102.6	....	....
28	(266,000)	103.9	(275,000)	107.4	(274,000)	107.1	(284,000)	111.0	....	....
29	(267,000)	103.9	(276,000)	107.4	(275,000)	107.0	(280,000)	108.9	....	....
30	(27,600)	120.3	(26,000)	113.4	(26,300)	114.7	(26,000)	113.4	....	....
31	(31,000)	108.1	(29,000)	101.1	(29,000)	101.1	(29,000)	101.1	....	....
32	(40,000)	110.8	(36,000)	99.8	(37,600)	104.2	(37,000)	102.5	(36,000)	99.8
33	(9,200)	93.7	(9,500)	96.8	(9,800)	99.8	(9,700)	98.8	(9,600)	97.8
34	(10,300)	93.6	(10,600)	96.4	(11,000)	100.0	(10,800)	98.2	(10,700)	97.3
35	(18,000)	99.2	(17,500)	96.4	(18,000)	99.2	(18,500)	101.9	(18,000)	99.2
36	(30,000)	99.8	(29,000)	96.5	(30,000)	99.8	(31,000)	103.1	(30,000)	99.8
37	(468,000)	113.2	(484,000)	117.0	(479,000)	115.8	(482,000)	116.6	(480,000)	116.1
38	(575,000)	115.0	(576,000)	115.2	(570,000)	114.0	(585,000)	117.0	(580,000)	116.0
39	(570,900)	110.1	(572,500)	110.4	(580,000)	111.9	(599,000)	115.5	(593,000)	114.4
40	((579,000))	111.8	((580,000))	111.9	((584,000))	112.7	((603,000))	116.4	((595,000))	114.8
41	(576,500)	110.1	(578,000)	110.4	(586,000)	111.9	(605,000)	115.5	(601,000)	114.8
42	(638,000)	109.6	(640,000)	110.0	(648,000)	111.4	(668,000)	114.6	(660,000)	113.4

AND ESTIMATED DISCHARGES

43	(55,600)	138.3	(49,000)	121.8	(47,000)	116.9	(48,000)	119.4	(50,000)	124.3
44	(35,000)	124.1	(33,000)	117.0	(32,000)	113.5	(33,000)	117.0	(34,000)	120.6
6	(99,000)	123.8	(104,000)	130.0	(106,000)	132.5	(104,000)	130.0	(102,000)	127.5
13	(72,000)	135.1	(74,000)	138.8	(70,000)	131.3	(68,000)	127.6	(66,000)	123.8
29	(267,000)	103.9	(276,000)	107.4	(275,000)	107.0	(285,000)	110.9	(282,000)	109.8
32	(40,000)	110.8	(36,000)	99.8	(37,600)	104.2	(37,000)	102.5	(37,000)	102.5
45	(568,600)	114.9	(572,000)	115.6	(567,600)	114.7	(575,000)	116.2	(571,000)	115.0
46	−6,400		−4,000		−2,400		−10,000		−9,000	
38	(575,000)	115.0	(576,000)	115.2	(570,000)	114.0	(585,000)	117.0	(580,000)	116.0
47	−1.3		−0.8		−0.5		−2.0		−1.8	

[e] Based on all available hydrographs, rainfall records, and other hydrologic data, 1817 to 1938, together with most authoritative evaluations of yearly discharge from Humphreys and Abbot's report and later analyses, supplemented by official discharge measurements for recent years.

[f] Uncorrected quantities derived from the gagings of headwaters and principal tributaries together with estimates for the intervening ungaged areas are enclosed in double parentheses; thus: ((583,814)).

errors not readily detected or controlled, it appears questionable whether the more tedious and detailed methods are justified when comparable results may be had by recourse to runoff coefficients, where in practice the consistency or inconsistency is made plainly apparent along with results.

Perhaps the main divergence of opinion herein expressed has to do with the statement in the final sentence of the "Synopsis" of the paper to the effect that the method is generally applicable to all drainage basins. The writer takes no exception to the use of the most detailed and exacting procedures that can be utilized to advantage in urban or other important or strategic areas where both damages and benefits may be of great magnitude and the necessary basic data from many stations are available. However, to attempt the expansion strictly along those same lines of procedure to cover all drainage basins seems to be an attempt at covering targets entirely beyond effective range of small-caliber weapons. If resort must be had to other devices, there can be no

TABLE 8.—Extension of Table 4, City Block 4841, St. Louis, Mo.

Storm No.	Duration (min)	Mean rainfall intensity (in. per hr)	Total Depth: Rainfall (in.)	Total Depth: Runoff (in.)	Runoff: Total (%)	Runoff: Maximum: %[a]	Runoff: Maximum: In. per hr	Storm No.	Duration (min)	Mean rainfall intensity (in. per hr)	Total Depth: Rainfall (in.)	Total Depth: Runoff (in.)	Runoff: Total (%)	Runoff: Maximum: %[a]	Runoff: Maximum: In. per hr
1	60+	1.29−	1.29	0.07	5	112	1.45	7	15	2.52	0.63	0.21	33	40	1.00
2	30	1.28	0.64	0.40	63	117	1.50	8	60+	1.30−	1.30	0.62	48	150	1.95
3	20	2.73	0.91	0.36	40	57	1.55	9	40	2.65	1.77	0.50	28	60	1.60
4	20	2.31	0.77	0.17	22	67	1.55	10	50	4.21	3.51	0.51	14	58	2.45
5	60+	1.14−	1.14	0.49	43	118	1.35	11	50	1.63	1.36	0.44	33	92	1.50
6	10	4.68	0.78	0.24	31	35	1.65	12	15	4.52	1.13	0.24	21	40	1.80

[a] Percentage of mean rainfall rate.

question that these will include coefficients expressing the percentage of rainfall appearing as runoff, either total volume, peak discharge, or average discharge for the critical periods of both storm and flood.

Even where plentiful records are available for rainfall, temperature, soil moisture, surface and cover conditions, infiltration, and all other essential factors capable of influencing runoff, it is well to keep in mind a timely caution from one of the most authoritative technical reports yet published.[29]

> "* * * Because of the existence of impervious or thinly mantled areas most rains, however light, produce some surface runoff, and to this extent the usefulness of the average infiltration capacity as a relative measure of the soil of a given basin to dispose of rainfall is diminished."

Table 4, presenting the results of gagings on a city block in St. Louis, together with the authors' analysis of the actual water losses that occurred for a dozen separate rains, may be extended to show the derived and associated data given in Table 8.

[29] "Floods of Ohio and Mississippi Rivers, Jan.–Feb. 1937," *Water Supply Paper No. 838*, U. S. Geological Survey, pp. 491–492.

It is significant that the records show variations in mean rainfall intensity for the separate storm periods from 1.14 to more than 4 in. per hr, with storm duration ranging from 10 to more than 60 min, with total depths ranging from 0.63 in. to 3.51 in., and runoff yields varying from 5% to 63% of the rainfall, whereas the "peak rate of runoff from area tributary to alley, in inches per hour," according to the final column of Table 4, averaged 1.62, with extreme variations from 1.00 to 2.45 in. Furthermore, from casual examination, the basic data presented for storms Nos. 1 and 10, Table 8, seem to be so irregular as to warrant review. It is difficult to accept such high infiltration, retention, and peak discharge rates for the same events, depending upon either 5% or 14% runoff coefficient, to produce the momentary peak discharges representing 1.45 and 2.45 in. per hr, respectively. Comparison with the adjacent storms

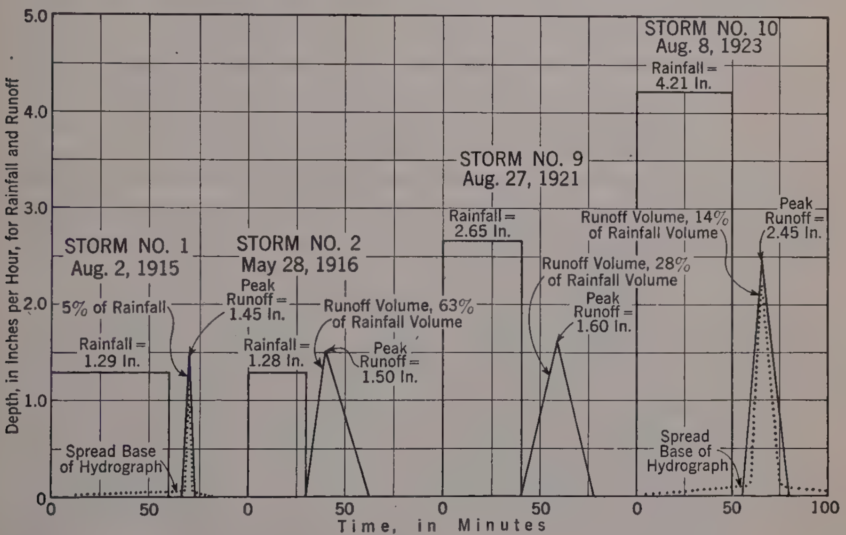

FIG. 15.—DIAGRAMMATIC REPRESENTATION OF RAINFALL AND RUNOFF VOLUMES AND RATES FOR SELECTED STORMS AT ST. LOUIS, MO., DATA FOR STORMS NOS. 1 AND 10 APPEARING TO BE QUESTIONABLE

Nos. 2 and 9, Table 8, nearly equal in mean rainfall intensity or in volume of storm runoff, as shown diagrammatically in Fig. 15, indicates the improbability of such runoff phenomena from the given rainfall. If the time bases of storms Nos. 1 and 10 were extended to cover the rainfall duration or double that period, then (maintaining the same volumes and maxima in accordance with the dotted hydrographs) the runoff peak would necessarily become more needle-like, and would challenge the reliability of the basic data, provided the underlying concepts and procedure of the unit hydrograph are valid. Conceding the fact that the maximum intensity may be much greater than the mean for a storm period, the writer is still unable to accept the 5% and 14% runoff yields of storms Nos. 1 and 10, in association with the peak rates of Table 4.

It is suggested that the only items inclining to show stability or a definite trend among the quantities of Table 4 are in the final column, and that for those twelve notable storms the average runoff rate might be assumed at its

average value with more assurance than may be accorded many of the assumptions underlying the procedure developed by the authors, especially if widespread use is contemplated, apart from their special guidance.

Examination of Table 5, summarizing the results of painstaking studies, drawings shown as Figs. 5, 6, and 7, and detailed analysis by the authors disclose that the mass supply, or rainfall minus infiltration of item 10, Table 5, persistently represents nearly 76% of the total rainfall, varying from 72% to 80% for both the 20-min and 60-min durations, and from 72% to 77% for the 40-min duration, all depending upon the rainfall pattern (that is, whether the intensity is uniform, advanced, intermediate, or delayed, according to the authors' definitions). In view of their admission immediately following Table 5, that the values there used "result in runoff rates greater than would be expected with the frequency involved," it seems almost an unnecessary refinement in general practice to forecast or choose the particular rainfall pattern, when any one of the four types would give results within 5 or 6% of the mean, and the mean runoff coefficient, 76% (which the authors propose to discard), could be so readily altered to conform more closely with the frequency involved, instead of retracing intricate paths by the procedure they have outlined.

The writer has repeatedly gone on record to the effect that the microscopic approach, unduly emphasized, has contributed to the general neglect of the great mass of hydrologic data, and of their readily recoverable values. For example, it has been shown that, in general, the average daily precipitation on days of rainfall (the average annual rainfall divided by the average number of rainy days per year) is a fair approximation to the 3-min maximum rainfall depth at many stations with reasonably long records.[30] Nearly a straight-line

TABLE 9.—PRECIPITATION IN INCHES, OBSERVED AT ST. LOUIS, MO., FOR VARIOUS PERIODS OF TIME WITHIN THE 104-YR PERIOD OF U. S. WEATHER BUREAU RECORD, INCLUDING 1940

MAXIMUM AND NEXT THREE VALUES						YEARLY		
3-min	5-min	15-min	60-min	24-hr	Month	Minimum and next three values	Mean	Maximum and next three values
(0.36)	0.59	1.39	3.51	7.02	17.07	23.23	39.23	68.83
(0.34)	0.56	1.19	3.47	5.08	11.43	23.38	....	65.36
(0.33)	0.55	1.03	3.36	4.80	11.26	24.80	....	61.40
(0.31)	0.51	0.92	2.42	4.19	10.90	25.00	....	52.72

relationship on logarithmic paper seems to connect the 5-min, 15-min, 60-min, and daily maximum precipitation, whereas the maximum monthly depth naturally exceeds the maximum daily, and in humid areas often closely approaches the minimum yearly precipitation. Armed with those facts, it is possible to make fair evaluations of maximum hourly or other short-period rainfall intensities to be expected for given frequencies.

Thus the readily available records for St. Louis, Mo., yield the items listed in Table 9. Now, assume that the maximum storm intensities for 24 hr, 60 min,

[30] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 398; also Fig. 4.

and lesser periods are desired, corresponding to frequencies of 104 yr or other period, or for one half, one third, or one fourth of such periods. They are found in Table 9 in the same order, beginning with the top items.

Rainfall depths for intermediate periods may be derived from their approximately straight-line relationship on logarithmic paper. For any assumed storm pattern with intensities high enough to produce notable runoff, it would seem evident that a fair approximation to the resulting hydrograph could be obtained for city block 4841 in St. Louis, Mo., as follows: By using a time base for the flood rise equal to two or three times the rainfall duration, up to an hour or perhaps two hours of storm; and, by adopting as a first approximation to the maximum ordinate, from 1 to 2 in. per hr (depending on antecedent conditions), subject to such adjustment as shall accommodate from 40 to 70% of the rainfall volume under the outflow hydrograph, and in general accordance with the observed data or derivations from Table 4. A satisfactory approach to estimated runoff may thus be made by the use of coefficients with a minimum of assumption and computations and the maximum support from characteristic behavior of the drainage area, as well as neighboring basins.

It seems to be more reasonable, if not more scientific, to accord due consideration to observation, experience, and the readily understandable empirical expressions that summarize such experience, to serve as controls and safeguards in general on all new methods and technical procedures designed to advance the frontiers of scientific knowledge and achievement.

G. W. Musgrave,[31] Esq.[31a]—If the authors of this interesting paper are successful in doing no more than convincing the great body of technical workers in the field of hydrology of the first point that they make—namely, the fallacy of using runoff coefficients—it is believed by this observer that a genuine service will be rendered. That runoff coefficients are a misconception has been shown by several workers, including LeRoy K. Sherman, M. Am. Soc. C. E., who has noted that runoff is a difference, not a ratio. Despite the evidence that has been presented, this writer feels that the use of this misconception will not immediately be discontinued. For this reason it is desired to present, by way of emphasis, a little additional data, which may be regarded as a more or less random sample of much similar evidence from other locations.

At the Conservation Experiment Station at Clarinda, Iowa, runoff records have been made for each storm of record since 1932, and for a large number of areas differing in treatments, sizes, slopes, or vegetal covers. In the data cited the runoff measurements are accurate to within 0.01 in. (more probably ± 0.001 in.) and rainfall is recorded by a number of Freiz, Weather Bureau, and special types of rain gages. Table 10 gives the rainfall and runoff for a 3-yr sample period from two areas having similar soil and slope, but differing in vegetal cover.[32] On one of these plots corn is grown annually and on the other bluegrass. It will be noted from the table that the average runoff from the corn for this period was 17.45% and from the bluegrass 2.03%. However,

[31] Head, Infiltration Studies, Office of Research, SCS, U. S. Dept. of Agriculture, Washington, D. C.

[31a] Received by the Secretary August 9, 1941.

[32] "Soil and Water Conservation Investigations," by G. W. Musgrave and R. A. Norton, *Technical Bulletin No. 558*, U. S. Dept. of Agriculture, February, 1937.

the range in magnitudes is not greatly different, being from 0 to 62.45% for the area growing corn and from 0 to 61.45% for the area in bluegrass. For the area growing corn, fifteen storms produced runoff in excess of 14% of the rainfall. For the area in bluegrass only two storms produced as much runoff as 14% of the rainfall. If one were using runoff coefficients, what value could be selected that would represent each of these two areas satisfactorily? It is obvious that the use of any single percentage to represent the runoff that might result from a given storm on either of these areas would be entirely fallacious.

The authors have indicated a number of the factors that affect infiltration capacity. In connection with the reference to soil porosity, it may be well to mention that it is probable that the total volume of porosity does not affect the rate of infiltration so much as does the pore-size distribution. This is obvious from a consideration of the soil porosity of a fine-grain soil such as clay, which in total volume of porosity usually is relatively large, but in which the infiltration rate is usually low because of the small average size of the individual pores.

In reference to surface slope, also, it may be well to indicate that, although existing evidence indicates little effect of degree of slope upon infiltration, there is evidence indicating that the effect of length of slope commonly is appreciable.

In referring to the variation in initial rates, the authors do not attribute the cause solely to soil-moisture variation, but properly take the broader view and consider "soil condition." To this also might be added the temperature of soil and of water. The effect of antecedent rain upon infiltration is a matter that is of great interest to hydrologists at the present time. Although its effect has been considered by some workers solely as that of filling the voids of a soil profile with water, the authors avoid this misconception. Clearly, the antecedent rain does much more to soil and soil structure than fill the

TABLE 10.—PERCENTAGE RUNOFF FROM PLOTS AT SOIL CONSERVATION EXPERIMENT STATION, CLARINDA, IOWA

Date	Rainfall (inches)	Annual Runoff, Plot 3; Corn		Runoff, Plot 8; Bluegrass	
		Inches	%	Inches	%
(1933)					
Mar. 31	0.52	0.034	6.54	0.000	0.00
May 12	0.82	0.030	3.66	0.000	0.00
May 22	0.29	0.000	0.00	0.000	0.00
June 27	2.88	0.646	58.73	0.091	3.16
June 29	1.10	0.234	8.13	0.000	0.00
Aug. 21	3.70	0.088	2.38	0.097	2.62
Sept. 25	1.76	0.107	6.08	0.000	0.00
(1934)					
May 12	1.30	0.740	56.92	0.000	0.00
Aug. 31	1.15	0.116	10.09	0.000	0.00
Sept. 3	1.22	0.063	5.16	0.000	0.00
Sept. 15	0.84	0.160	19.05	0.000	0.00
Sept. 26	1.73	0.493	28.50	0.000	0.00
Oct. 19	2.37	0.824	29.19	0.106	4.47
Oct. 20	0.37	0.108	34.77	0.000	0.00
Nov. 3	1.39	0.214	15.39	0.000	0.00
Nov. 22	0.47	0.043	9.15	0.000	0.00
(1935)					
Jan. 19	0.53	0.331	62.45	0.326	61.51
May 21	2.22	0.000	00.00	0.000	0.00
May 23	0.73	0.109	14.93	0.000	0.00
May 28	1.55	0.285	18.39	0.000	0.00
June 1	1.57	0.667	42.48	0.000	0.00
June 3	0.97	0.377	38.87	0.000	0.00
June 4	0.47	0.138	29.36	0.065	13.83
June 10	0.30	0.037	12.33	0.000	0.00
June 24	0.79	0.228	57.54	0.000	0.00
June 26	0.61	0.351	28.86	0.000	0.00
Sept. 2	1.84	0.000	00.00	0.000	0.00
Sept. 16	0.42	0.058	13.81	0.093	22.14
Sept. 26	1.98	0.100	5.05	0.000	0.00
Oct. 17	1.28	0.000	0.00	0.000	0.00
Oct. 31	1.10	0.098	8.91	0.000	0.00
Total	38.27	6.679	....	0.778	....
Average	....	....	17.45	....	2.03

voids with water. The impact of rain drops is such as to apply a large amount of energy to the surface of the soil. During 1941, J. O. Laws[33] provided the necessary basic data upon which the amount of energy of impact may be calculated, and it is apparent that much of the destruction of aggregated soil structure at the surface can be accounted for readily by this ample source of energy.

In addition to these mechanical effects, of course, there are many other effects of antecedent rains. There may be an inwashing of fine particles that lodge in constricted portions of the pores and channels of the upper part of the profile. There is likely to be swelling not proportional to the quantity of water. Also, with the withdrawal of moisture upon drying, the soil does not necessarily return at once to its former structural or permeability status. Actually, direct relations between the content of soil moisture and the amount of subsequent infiltration may be expected only under certain unusual conditions, such, for example, as unigranular sands lying upon impermeable substrata—conditions in which the only effect of water is that of filling voids.

At the Edwardsville project of the Soil Conservation Service, which has been mentioned by the authors (Fig. 4), data are available that include repeated infiltrometer runs on a series of ten plots distributed over the watershed. These data include soil moisture at three different depths; and the temperature of soil, air, and water; as well as other environmental conditions. The data on soil moisture and soil temperature for 1940 are given in Table 11.

TABLE 11.—COMPARISON OF INFILTRATION (f_c VALUES), WITH ANTECEDENT SOIL MOISTURE FOR THREE DEPTHS, AND SOIL TEMPERATURE DURING REPEATED INFILTROMETER WET RUNS AT THE EDWARDSVILLE, ILL., WATERSHED

Plot	Date (1940)	Infiltration, f_c, in in. per hr	Soil Moisture (%)			Soil temperature (degrees F)
			0 in. to 7 in.	7 in. to 14 in.	14 in. to 21 in.	
7	May 7	0.25	27.3	28.7	25.4	64
7	June 25	0.41	22.9	21.1	17.4	68
7	July 29	0.45	24.8	23.2	20.7	78
7	Sept. 18	0.25	24.4	23.2	21.8	71
8	May 9	0.23	26.4	25.8	24.4	59
8	June 26	0.39	22.3	19.1	20.1	74
8	Aug. 1	0.42	22.3	22.2	20.0	78
8	Nov. 7	0.24	23.0	21.3	19.8	49
9	May 23	0.06	25.4	24.2	23.3	62
9	June 27	0.18	25.5	16.9	21.0	73
9	Aug. 2	0.31	26.4	22.6	22.0	77
9	Sept. 23	0.34	25.8	22.0	22.5	70
10	May 25	0.04	29.0	25.5	26.2	62
10	June 29	0.04	27.1	27.1	27.7	65
10	Aug. 5	0.07	29.0	23.5	26.2	72
10	Nov. 29	0.18	28.2	26.5	26.5	46
11	June 6	0.02	30.6	28.7	27.5	74
11	July 1	0.17	30.5	32.3	27.2	68
11	Aug. 7	0.07	28.7	29.2	25.0	76
11	Sept. 30	0.08	28.6	29.8	26.0	60
12	June 5	0.04	20.2	26.9	26.0	71
12	July 2	0.12	29.6	28.5	26.6	64
12	Aug. 9	0.00	30.8	30.2	27.6	76
12	Nov. 20	0.05	31.9	29.0	25.6	52
13	May 27	0.00	26.2	25.9	25.4	60
13	July 8	0.25	27.2	26.0	25.2	74
13	Aug. 13	0.22	27.3	26.0	25.4	76
13	Oct. 7	0.20	26.8	26.3	25.6	57
14	May 28	0.34	28.0	24.4	23.5	60
14	July 9	0.50	25.5	21.8	18.6	76
14	Aug. 15	0.28	25.3	25.7	21.9	78
14	Dec. 9	0.13	26.1	27.4	23.3	40
15	June 3	0.27	24.4	24.8	25.7	69
15	July 10	0.24	18.0	16.6	19.0	78
15	Aug. 19	0.185	23.1	18.9	18.5	62
15	Dec. 20	0.05	25.4	27.7	25.2	38
16	June 4	0.40	28.9	27.9	27.3	69
16	July 12	0.35	26.2	26.2	25.1	63
16	Aug. 20	0.10	20.5	22.7	21.6	64
16	Oct. 11	0.12	22.0	19.0	19.4	60
Mean		0.20	26.0	24.9	23.7	65.8

It will be seen that repeated runs on the same plot do not give infiltration

[33] "Measurements of the Fall-Velocities of Waterdrops and Raindrops," by J. O. Laws; publication pending in *Transactions*, Am. Geophysical Union, 1941.

f_c values (the minimum rate of infiltration occurring during the last part of the wet runs) that invariably correlate with soil moisture. More detailed analyses of the data, however, throw a little more light upon the subject. Four successive rounds of determinations were made on each of the ten plots, and the means for each round are presented in Table 12. Here it is seen that

TABLE 12.—COMPARISON OF MEANS OF SOIL MOISTURE, SOIL TEMPERATURE, AND INFILTRATION FOR EACH OF FOUR ROUNDS OF DETERMINATIONS ON TEN PLOTS AT EDWARDSVILLE, ILL. (WET RUNS)

Round	Period (1940)	Infiltration, f_c, in in. per hr	Soil Moisture (%)			Soil temperature (degrees F)
			0 in. to 7 in.	7 in. to 14 in.	14 in. to 21 in.	
1	May 5 to June 6	0.165	26.64	26.28	25.47	65.0
2	June 25 to July 12	0.265	25.48	23.56	22.79	70.3
3	July 29 to Aug. 20	0.210	25.82	24.42	22.89	73.7
4	Sept. 18 to Dec. 20	0.164	26.22	25.22	23.57	54.3

infiltration for rounds 2 and 3, which were made during the summer period, is higher than for rounds 1 and 4, during the spring and fall, and that the average soil-moisture content at the 14-in. to 21-in. depth was lower for rounds 2 and 3 than for rounds 1 and 4. However, it is seen also that the temperature of the soil was higher during the summer period, as would be expected. This quadratic or second-degree curve of seasonal trend of infiltration with changing moisture and temperature is statistically significant.

An examination of the results in Table 12, without due allowance for known differences such as time, might readily lead to erroneous conclusions since they appear to show a very strong inverse correlation between moisture and temperature, as well as between moisture and infiltration, and also a strong positive correlation between temperature and infiltration. More detailed analyses of the individual determinations, however, show that the apparent relationship between moisture and temperature is not significant, that the inverse relation of moisture and infiltration is fairly marked for rounds 2 and 3 but not for rounds 1 and 4, and that for the entire series of forty determinations moisture and temperature in combination are strongly associated with infiltration.

One must recognize, however, the strong probability that other variables are involved and the possibility that the rate of infiltration is actually dependent primarily upon such factors. For example, it is known that the pore-size distribution of a soil varies and that it well may be least in the spring and fall and highest in the summer—in other words has the same trend as the seasonal infiltration curve. In these data there are indications not only that changes in both temperature and moisture are associated with changing infiltration, and that under certain conditions one of these variables may appear more important than the other, but also that at the present time the hydrologist may be measuring merely "shadows" of the true causal factor—pore-size distribution.

This example is fairly typical of the effect of soil moisture as it has been found in many studies covering the period 1929 to date (1941). Briefly, it may be stated that ordinarily:

(a) Seasonal trends of soil moisture and soil temperature tend to be correlated inversely;
(b) Infiltration is lower when soil moisture is higher, but the magnitude of reduction is not proportional to the increase in moisture;
(c) Subsoil moisture often shows a more marked effect upon infiltration than does soil moisture in the surface;
(d) Seasonal trends (similar to those of Table 12) ordinarily are statistically significant; and
(e) There are continuing evidences that other factors in addition to that of soil moisture are exerting their effects upon the rate of infiltration.

Looking at the matter from a purely physical basis, there seems to be no reason why one should commonly find mathematical proportionality relations between soil moisture and infiltration, since obviously soil moisture alone is not an adequate basis for expressing the size of soil pores. Perhaps the additional variables in the complex may be identified and measured during the course of some of the many investigations now under way.

Messrs. Horner and Jens have been making studies of rainfall and runoff for some time and have had opportunity to test, under many conditions, the analytical procedures that they have presented. In their example of a rational procedure without the use of runoff coefficients, they have presented material that should be distinctly useful. The writer agrees with them wholly in principle, and merely suggests caution against oversimplification in the handling of some of the variables, the effects of which are undoubtedly highly complex, and the magnitude and effect of which are only now being investigated.

F. L. Flynt,[34] Assoc. M. Am. Soc. C. E.[34a]—In their detailed analysis of storm-water runoff from typical urban areas, the authors have demonstrated beyond reasonable doubt that it is more rational to consider runoff as rainfall minus losses than it is to assume that runoff may be expressed as a constant percentage of the rainfall for a given storm. They have also demonstrated that it is practicable to use this principle in the design of drainage systems. The application of the equations for overland flow and the routing of the resulting hydrographs through alley and gutter storage introduce a new technique in the analysis and synthesis of runoff from urban areas.

It is evident that the refinements introduced into the design procedure, through a more complete understanding of the fundamental principles involved, tend to increase the work required to design a given drainage system, but this is not a valid argument against the use of the improved method. It would be just as logical to ignore secondary stresses in the design of large framed structures or to provide for them by an arbitrary increase in the factor of safety.

[34] Associate Engr., U. S. Engr. Dept., Rock Island, Ill.

[34a] Received by the Secretary August 14, 1941.

The old saying "Anything worth doing at all is worth doing well" applies to the design of engineering structures as well as to lay activities.

Perhaps the weakest link in the chain of the demonstration is the first of the six essential stages of design as listed in Part I under "Application to Hydraulic Engineering"—namely, the delineation of the precipitation pattern. This weakness is not inherent in the method but is due to the present lack of information that further research may some day supply. So far as the writer is aware, no determinations have been made of the relative frequency of various types of storm patterns, such as "advanced," "intermediate," and "delayed," which would enable a satisfactory choice to be made, appropriate to a given design frequency. Fortunately, the effect of rainfall pattern upon the maximum rate of runoff is less apparent in the hydrograph of outflow from a drainage network serving a large area, where the time of concentration is comparatively long, than it is for a single block. This is made clear from a study of Figs. 5, 6, and 7.

The detailed method of analyzing the hydrograph of runoff from urban areas, as demonstrated in the paper, is equally applicable to the problem of designing drainage structures to carry surface runoff from airport areas. Such an analysis, when the grading layout is closely coordinated with it, enables a balanced design to be made that not only assures an adequate drainage system, but also one which is the most economical for the given conditions.

In the analysis of stream-flow hydrographs, the fundamental principle of considering runoff as rainfall minus losses rather than as a ratio or coefficient is rapidly coming into general use. The methods of analysis in the case of drainage areas, involving many square miles, of course, are quite different in detail from the examples given in the paper. The reasons for this difference may be outlined as follows:

(*a*) In the case of streams the volume of water in storage as overland flow or surface detention is usually quite small when compared with water in channel storage.

(*b*) Time periods of hours, rather than minutes, must be used, which means that loss rates must be averaged not only over large areas but also over comparatively long periods of time. This makes it more difficult to relate the observed loss rates with the infiltration capacities of the various soils in the area.

(*c*) The infiltration rate for a stream basin varies over a wider range than it does for urban or airport areas. This is due to the fact that in agricultural areas the condition of the top layer of soil is modified from season to season and from year to year through cultivation and changes in vegetal cover.

(*d*) Because of the present lack of an adequate method of segregating the various kinds of water loss, the experimental values of loss rate as determined from a comparison of rainfall and runoff volumes do not, necessarily, reflect the true value of infiltration loss but the aggregate of all losses, whatever their nature.

(*e*) Another disturbing factor, common to stream basins but insignificant in urban areas, is ground-water return to the flood hydrograph. The problem

of segregating the ground-water and surface-water hydrographs has not yet been completely solved. This is partly because there is often no sharp line of demarcation between the two. Some of the ground water travels underground only a few feet before appearing in stream flow, whereas some of it travels through subsurface strata for great distances.

(*f*) Uniform rainfall over small urban areas may be assumed, but for large areas uniform rainfall of significant intensity is practically unknown. The areal distribution of rainfall becomes increasingly important as the size of the watershed increases. Cases have been observed in which appreciable abnormalities in the shape of runoff hydrographs have been traced to heavy rainfall which had been unofficially observed over limited areas, but which had not been recorded at any Weather Bureau Station.

The effects of all the aforementioned conditions, which tend to complicate the rainfall-runoff relation, become more pronounced with the increase in size of the drainage area. It follows that, for the purpose of analyzing hydrographs of record and for synthesizing design floods, it is advisable to subdivide the drainage area into sub-basins and to subdivide the rainfall into periods as short as practicable, noting the areal distribution of rainfall for each period over each sub-basin. By the use of synthetic unit graphs where no hydrographs are available, and by routing and combining the hydrographs of the various sub-basins, hydrographs of past floods may be reconstituted with reasonable accuracy from the rainfall data, and design floods may be synthesized from assumed or transposed rainfalls with more confidence than if large basins were considered as a unit. The amount of work involved in this procedure is considerably greater, of course, than when the older methods are used; but the more reliable results justify the increased work.

Thus it is seen that the principle of breaking down the area tributary to the point of measurement into simpler components for the purpose of analysis and synthesis of runoff hydrographs is of general application and, no doubt, will eventually become the standard practice as will the evaluation of runoff as rainfall minus losses instead of in terms of the percentage factors formerly used.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

SALTS IN IRRIGATION WATER

Discussion

BY MESSRS. HERMAN STABLER, AND M. R. LEWIS

HERMAN STABLER,[4] M. AM. SOC. C. E.[4a]—Too few engineers are sufficiently interested in quality of water to delve into the mysteries of water chemistry, and too few chemists are sufficiently interested in the engineering applications of the water analyses they make to consider the impact of quality of water on industrial and agricultural development. The author has done a service to the engineering profession by invading that twilight zone between engineering and chemistry and bringing back and presenting to engineers a classification of waters in terms of their chemical constitution and an ingenious method of expressing chemical quality and classification in graphical form.

In general, the author follows the Palmer method of considering waters as a balanced system of chemical values. Palmer[5] advocated the statement of a water analysis in four parts:

(1) The properties of the solution in percentage proportions (in which he grouped radicals to indicate "properties," of which there would be but three to any water; properties so determined fixed the "class" of the water);

(2) The percentage reacting values of groups of radicals accompanied by a statement of concentration values;

(3) The "character formula"—the percentage reacting values of the individual radicals—together with a statement of concentration values; and

(4) The base analyses.

Palmer's designation of the "class" of a water, or of the "properties" that determine the class, provides a generalized chemical picture of the water that is scientifically correct and convenient for some purposes but is inadequate for industrial or agricultural evaluation because it recognizes no difference in effect of the individual radicals grouped. The author departs slightly from the

NOTE.—This paper by Raymond A. Hill, M. Am. Soc. C. E., was published in June, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1941, by C. S. Scofield, Esq.

[4] Chf., Conservation Branch, U. S. Geological Survey, Interior Dept., Washington, D. C.

[4a] Received by the Secretary August 28, 1941.

[5] "The Geochemical Interpretation of Water Analyses," by Chase Palmer, *Bulletin No. 479*, U. S. Geological Survey, 1911, p. 14.

Palmer grouping of radicals and states (see heading "Geochemical Classifications"): "For the interpretation of analyses of water used in agriculture it was felt desirable to group the sulfates with the carbonates rather than with the chlorides and nitrates." He thus violates the chemical logic of the Palmer grouping without seeming to gain anything in application to irrigation engineering. The writer wonders why.

Palmer's character formula gives a detailed chemical picture of a water portraying the particular place it occupies in an infinite series of "classes." Many individuals of this infinite series are grouped in the few major classes based on "properties." Together with concentration value, the character formula can be applied to the solution of many industrial and irrigation problems. The author follows the principle of Palmer's character formula with the exception that his percentage reacting values (or equivalents per million) add up to 200 instead of 100. Here again the writer sees no gain. Calculating the percentages of "anions" and "cations" separately is desirable, but it is as easy to make the sum of each 50, thus preserving the concept of 100% for the whole, as to double the values as the author has done.

The author's diagrammatic representation of a water by a single point, or by designation (for example, as a Type II or Type *b*II*c* water), carries with it a degree of significance but leaves out of the picture rather important detail. Designation of Type *b*II*c* for the Verde River water, for example, means (following the author's definitions) that this is "a water in which no one of the positive ions exceeds 50% of the total" but "in which more than 50% of the anions are bicarbonates or carbonates," in which most of the salts fall in a group of Ca, Mg, SO_4, H CO_3, and CO_3 ions, some in a group of Na, K, SO_4, H CO_3, and CO_3 ions, and none in a group of Ca, Mg, Cl, and NO_3 ions (see heading "Types of Water"). All of this and more would be disclosed in Palmer's character formula which, for the same water, would be 21.6 Ca, 17.2 Mg, 11.2 Na, 36.1 H CO_3, 9.2 SO_4, 4.7 Cl, simulating the formula for any other mineral.

Diagrams showing characteristics of waters and adapted to comparison of several analyses have been developed by a number of people. The writer has been accustomed to use the form shown in Fig. 2, in which comparison is made of the nine waters in Table 3. This diagram shows, in parallel columns (instead of in two separate triangles as shown by the author), the factors entering into the Palmer character formula, and, in width of column (instead of area of circles as shown by the author), the concentration factor which, together with the character formula, makes up the complete detailed picture of a water analysis. The writer finds it easier to diagnose the character of a water and to compare the several characteristics of a group of waters from Fig. 2 than from the author's "geochemical chart" (Fig. 1), but recognizes that this may be largely if not wholly due to habit and his greater familiarity with the one than with the other. Both schemes accomplish essentially the same purpose, and choice between them resolves itself largely into a matter of personal preference. Fig. 2 shows at a glance the variations from upper to lower reaches of the river, which are:

(1) Progressive increase in concentration until at Fort Quitman concentration is nearly thirty times as great as at Del Norte;

(2) Progressive decrease in the percentage of Ca from Del Norte to Fort Quitman interrupted by a maximum at Otowi;

(3) Progressive decrease in the percentage of Mg;

(4) Progressive increase in the percentage of Na + K interrupted by a minimum at Otowi;

(5) Progressive decrease in the percentage of CO_3 + $H\,CO_3$ interrupted by a sub-maximum at Otowi;

(6) Progressive increase in the percentage of SO_4 from Del Norte to Elephant Butte (except for slight decrease at Otowi), followed by progressive decrease downstream; and

(7) Progressive decrease of the percentage of Cl from Del Norte to Otowi, followed by a rather sudden increase to San Marcial and Elephant Butte and then by progressive increase downstream.

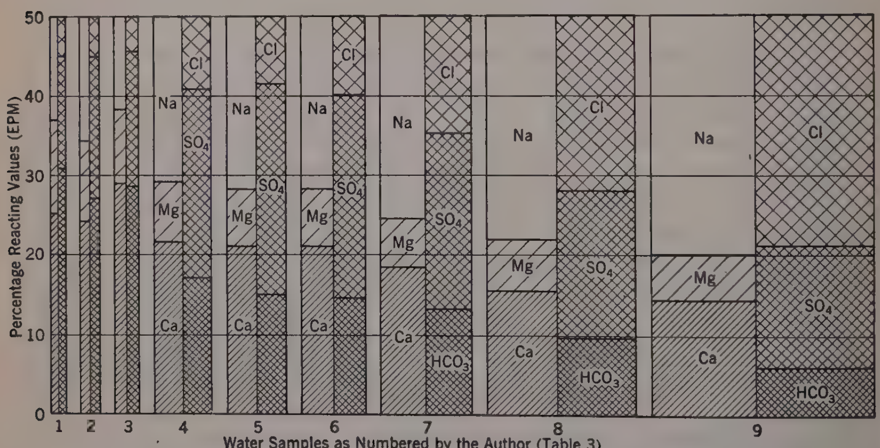

FIG. 2.—GRAPHICAL REPRESENTATION OF "CHARACTER" FORMULA (PALMER) AND CONCENTRATION VALUE FOR WATERS OF THE RIO GRANDE (WIDTH OF COLUMNS PROPORTIONAL TO CONCENTRATION, COL. NO. 1 BEING DOUBLE SCALE)

These variations can likewise be visualized from the author's anion and cation triangles in the geochemical chart in Table 3 or from a table of radicals by reacting values and concentration values (Palmer's third item in the statement of an analysis). The changes in "class" shown in the author's diamond in the geochemical chart are progressive increase (except for Otowi) in percentage hardness, progressive decrease (except for Otowi) in percentage saltness, and general progressive decrease in percentage alkali to Elephant Butte, followed by progressive increase downstream. Do these add anything of importance in evaluating an irrigation water? The writer thinks not. Had the author adhered to the Palmer grouping for properties there would be: A general progressive decrease of secondary alkalinity (beneficial or inert for irrigation); progressive increase of secondary salinity (beneficial for irrigation) to Leasburg,

followed by decrease at El Paso and increase thereafter; and progressive increase (except at Otowi) in primary salinity (harmful for irrigation). The irrigation picture, however, would not be very well defined. In evaluating waters for industrial use or for irrigation the writer prefers to use formulas adapted especially to the particular problem in hand rather than to trust to any general geochemical classification.

The author does not attempt to describe the effect of different kinds of water on soil and on plant growth but expresses the hope that some discusser will do so. The writer has not been so cautious. In 1910[6] he prepared an article on "The Relative Value of Irrigating Waters" that was republished[7] one year later with modifications as part of a more comprehensive treatment of industrial interpretation of water analyses.

Irrigation waters were discussed, formulas were given for computing the "alkali coefficient" (an indicator of value for irrigation purposes), and a classification of irrigation waters was set forth. The treatment there accorded the subject of irrigation waters, though far from adequate, has proved very useful in certain phases of water evaluation by the writer. The data then available were meager, and more accurate formulas doubtless could be devised on the basis of the wealth of information and experience now available. This classification of irrigation waters includes consideration of only the common constituents reported in water analyses. Any adequate treatment of the subject would have to take into consideration such substances as boron which, although present in irrigation waters in small amounts, may exert a potent influence on plant life.

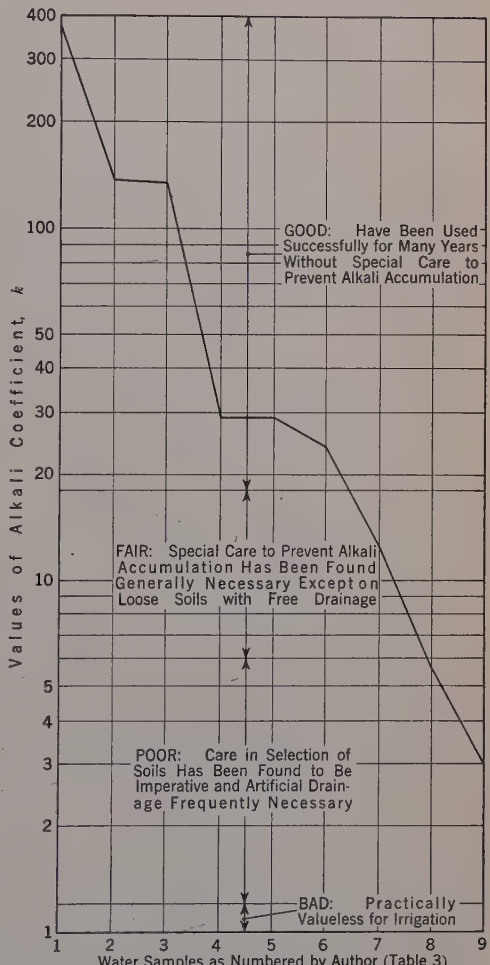

FIG. 3.—CLASSIFICATION AND RELATIVE VALUE FOR IRRIGATION OF WATERS OF THE RIO GRANDE

[6] *Engineering News*, Vol. 64, No 2, July 14, 1910, p. 57.

[7] *Water Supply Paper No. 274*, U. S. Geological Survey, Washington, D C., 1911 (including on p. 165 a chapter on "The Industrial Application of Water Analyses").

For what it is worth the writer presents in Fig. 3, by the method outlined in *Water Supply Paper No. 274*, on semilogarithmic scale, the alkali coefficient, class, and class description of the nine waters of the Rio Grande whose analyses are supplied by the author. The waters at and above Otowi are shown to be of very high class for irrigation purposes; those at San Marcial, Elephant Butte, and Leasburg are shown as "good" but little better than "fair"; the water at El Paso is shown as "fair"; and those below El Paso as "poor." How nearly do these classes and the relative magnitude of the alkali coefficients on which they are based reflect the experience of the water users along the Rio Grande?

In 1935 Carl S. Scofield[8] discussed the many "variables that must be considered in determining the relationship between the quality of irrigation water on the one side and of a profitable and enduring system of irrigation agriculture on the other." With a warning that the class limits or standards given are intended for use in a particular area and should not be taken as applicable everywhere, he presents the values in Table 5 as an example of the permissible

TABLE 5.—PERMISSIBLE LIMITS OF CLASSES OF IRRIGATION WATER

Class	Grade	Concentration, Total Dissolved Solids[a]		Percentage of sodium[b]	Boron, Parts per Million; Crops Group			Concentration in Milligram Equivalents	
		Conductance $K \times 10^5$ at 25°C	Parts per million		A	B	C	Chlorides (Cl)	Sulfates (SO_4)
1	Excellent	<25	<175	<20	<0.33	<0.67	<1.0	<4	<4
2	Good	25–75	175–525	20–40	0.33–0.67	0.67–1.33	1.0–2.0	4–7	4–7
3	Permissible	75–200	525–1,400	40–60	0.67–1.00	1.33–2.00	2.0–3.0	7–12	7–12
4	Doubtful	200–300	1,400–2,100	60–80	1.00–1.25	2.00–2.50	3.0–3.75	12–20	12–20
5	Unsuitable	>300	>2,100	>80	>1.25	>2.50	>3.75	>20	>20

[a] The concentration of dissolved salts in water may be measured by either of two methods—that of electrical conductance and that of evaporating the water and weighing the residue. [b] This percentage represents the proportion of sodium to the total cations and is computed from the data of analysis, reported as milligram equivalents, by dividing the sum of the values for sodium and potassium by the sum of the values for all the cations.

limits, adopted for a definite region, of classes of irrigation water with respect to certain of its characteristics.

Applying the standards of Table 5 to the nine Rio Grande waters whose analyses are submitted by the author (a procedure that Scofield would probably regard as incautious), the series of classifications in Table 6 is derived and compared with the writer's alkali coefficient and classification.

By either system the first two waters stand out as of high class; the third is equally good by the writer's classification but slightly poorer by Scofield's; the fourth rates about the same under the two classifications; the fifth and sixth, almost identical with the fourth according to the writer's classification, are slightly poorer by Scofield's; and the two classifications give about the same result on the eighth and ninth waters.

[8] "The Salinity of Irrigation Water," by Carl S Scofield, The Smithsonian Rept. for 1935, pp. 275–287.

In general, the two classifications should give similar results except with primary alkaline waters (a class not represented in the nine waters of the Rio Grande) for which the writer's classification would be substantially more severe. This is explained by a more lenient evaluation of carbonates by Scofield in the

TABLE 6.—Comparison of Classifications

Water No.	Scofield Classification				"Alkali coefficient"	Class
	Concentration	% of Na	Chlorides	Sulfates		
1	Excellent	Good	Excellent	Excellent	356	Good
2	Excellent	Good	Excellent	Excellent	139	Good
3	Good	Good	Excellent	Excellent	135	Good
4	Good	Permissible	Excellent	Good	29	Good
5	Permissible	Permissible	Excellent	Good	29	Good
6	Permissible	Permissible	Excellent	Good	24	Good
7	Permissible	Permissible	Good	Good	12.5	Fair
8	Permissible	Permissible	Permissible	Permissible	5.6	Poor
9	Doubtful	Permissible	Doubtful	Permissible	3.0	Poor

light of the experience of later years. Computation of the writer's alkali coefficients was developed under the older viewpoint and presupposes Na_2CO_3 twice as undesirable as Na Cl and ten times as undesirable as Na_2SO_4.

M. R. Lewis,[9] M. Am. Soc. C. E.[9a]—The importance, not only of the total quantity of dissolved salts, but also of the character of the salts is shown well by Mr. Hill. There is need for more general knowledge and consideration of both factors in irrigation practice.

In the Pacific Northwest there appears to be a close correlation between the pH-value of soils and soil solutions with the productivity and ease of reclamation of alkali lands. Herein and in common usage the term "alkali" includes both saline (white alkali) and alkaline (black alkali) soils. For this reason the carbonate and bicarbonate content appears to be of particular importance in agricultural practice. To the extent that this is true, it would be better to retain Mr. Palmer's method[2] of grouping the SO_4 ion with the Cl, rather than with the H CO_3 and CO_3 ions. Apparently, the alternative grouping is necessary in the development of the author's geochemical chart.

Mr. Hill's conclusion that some salts from the irrigation supply are retained in the valley and that a large part of the salts carried out with the drainage water come from the original ground water, and also his determination of the approximate character of both, seem well founded. His opinion, that the residual salts are held in the soil and not in the ground water and that the general result of irrigation in the area is adverse, seems open to argument. C. S. Scofield[10] has shown that ground water held near the surface of irrigated soils

[9] Senior Agri. Engr., SCS, U. S. Dept. of Agriculture, and Oregon State College, Corvallis, Ore.

[9a] Received by the Secretary September 5, 1941.

[2] "The Geochemical Interpretation of Water Analyses," by Chase Palmer, *Bulletin No. 479*, U. S. Geological Survey, 1911.

[10] "The Movement of Water in Irrigated Soils," by C. S. Scofield, *Journal of Agricultural Research*, Vol. 27, 1924, pp. 617–694.

does not move rapidly to the drainage channels. Is it not probable that a large part of the "residual" salts are held in the ground water below irrigated fields rather than in the soil of the root zone? Since these residual salts are of less harmful quality than either the influent salts or the displaced salts, it would appear that the net result of the changes may well be beneficial rather than harmful.

The evidence of base exchange activity in the soil is another matter. Truly more study is needed.

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DISCUSSIONS

CONSUMPTIVE USE OF WATER FOR AGRICULTURE

Discussion

BY MESSRS. S. T. HARDING, C. S. JARVIS, AND R. W. DAVENPORT AND GORDON R. WILLIAMS

S. T. HARDING,[67] M. AM. SOC. C. E.[67a]—Although the conditions which affect consumptive use are numerous and subject to much variation, the authors present records which indicate that the total useful heat supply may be the most important single factor. As the records of daily maximum temperatures are secured and published at Weather Bureau Stations in nearly all agricultural areas, the accumulated daily maximum air temperature above 32° F during the growing season can be computed for areas where estimates of consumptive use may be needed. Such estimates are frequently needed for western areas because the total irrigated areas that may be supported adequately by any water supply (where full recovery of conveyance and percolation losses can be made) are determined by the rate of consumptive use.

The authors use the heat supply for the growing season and the rainfall for the full year. This is consistent. Rainfall in the non-growing season will either appear as runoff and be taken care of in the outflow records or remain as soil moisture for use during the crop season. Consumptive use of rainfall in the non-growing season, as this is defined by the authors, will be relatively small and may be neglected. For many California areas the authors' definition of the growing season will include the entire year.

Table 1 separates results for irrigated valleys from those for non-irrigated watersheds. The latter are similar to the watersheds for which other computations of consumptive use have been published in *Water Supply Paper No. 846*.[68] The water losses (as this term is used in *Water Supply Paper No. 846*) are equivalent to the authors' definition of consumptive use, since adjustments

NOTE.—This paper by Robert L. Lowry, Jr., M. Am. Soc. C. E., and Arthur F. Johnson, Assoc. M. Am. Soc. C. E., was published in April, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1941, by E. B. Debler, M. Am. Soc. C. E.; and September, 1941, by Messrs. Rhodes E. Rule, Edgar E. Foster, Charles H. Lee, Harry F. Blaney, and J. L. Burkholder.

[67] Prof., Irrig., Univ. of California, Berkeley, Calif.

[67a] Received by the Secretary August 6, 1941.

[68] "Natural Water Loss in Selected Drainage Basins," by G. R. Williams and others, *Water Supply Paper No. 846*, U. S. Geological Survey, 1941.

were made for the loss from water areas. The values of the resulting mean annual water loss in *Water Supply Paper No. 846* for eighteen different parts of the drainage area of the Delaware River vary from 16.9 in. to 32.8 in. although the precipitation varied only from 92% to 110% of the mean of 44 in. These results are secured from more general records than those used by the authors and would be expected to vary more widely. The authors limited the records they used to areas which they considered to be fully supplied with moisture; the rainfall in the Delaware Basin should be sufficient to approach or exceed this standard. An application of the authors' method of comparison of effective heat and consumptive use to the results shown in *Water Supply Paper No. 846* would indicate the range of variation to be expected where only such general records are available.

The authors make a distinction between the consumptive use on agricultural and nonagricultural areas and reduce their results to the rate of consumptive use per acre of cropped area. Allowance when needed, both for areas using only rainfall and for areas of swamp or other excess rates of use, is essential if results applicable to crop areas are to be secured. The authors' procedure for such areas and the amount of the differences made for these lands are not consistent, however, with their treatment of all crops as having the same rate of consumptive use. They state that, notwithstanding the wide differences in crops in the areas used (mountain meadow hay and alpine forests to cotton and citrus fruits), the relation between consumptive use and effective heat is maintained closely. This should not be accepted too completely. The close relationship shown in Fig. 2 appears to be the result of balancing factors in the different areas rather than the uniformity of use by different crops. In several of the areas listed grain can be grown without irrigation with rainfalls less than the amount of the consumptive use shown. The differences in the moisture requirements of forage and orchard crops are also recognized in irrigation practice. The relationship shown in Fig. 2 would need to be modified when applied to local areas to allow for the types of crops to be grown.

The points plotted in Fig. 2 fall as consistently along the straight line used as can be expected in results of this type. This does not mean that this line can be extended outside of the range of these records, however. Its extension downward would indicate a consumptive use of about 1 ft in depth, per year, in an area having no effective heat. This is a probable value as yearly moisture losses on an area where the climate permits any growth of vegetation or evaporation from the soil would be expected to approach this amount. The authors' points 18 to 20 indicate that the extension of the curve upward should be on a reduced slope reaching a maximum consumptive use somewhat above 3 ft per yr.

Such a maximum consumptive use under large values for effective heat is also indicated by other records. The maximum values for effective heat in the United States are about 20,000 day-degrees F per yr. This value occurs on the lower Colorado River areas in Arizona and California. For such amounts of effective heat the extension of the line in Fig. 2 gives a consumptive use of about 4 ft per yr. This is about the average rate of delivery to the land of irrigation water plus rainfall in the Imperial Valley and exceeds the

similar figure for the Yuma Project. The delivery to the land in both of these areas includes some deep percolation losses not consumed by the crops so that the actual consumptive use will be less than that indicated by Fig. 2. This does not mean that the relationship in Fig. 2, within the limits of its supporting data, is incorrect; but it indicates that, in areas of greater effective heat, consumptive use should reach a maximum at or greater than 3 ft per yr. In such areas it is not customary for land to be fully cropped for the entire year and this difference is the result of not utilizing the full available heat supply.

The authors have included rainfall in the water supply from which the consumptive use is determined. This is desirable in results that may be used in both irrigated and nonirrigated areas. In some parts of California the rainy and irrigation season are sufficiently separate that values of the consumptive use may be derived which are applicable to the irrigation supply only. This practice is useful in these areas in determining the irrigation supply that needs to be provided but such results should not be applied to areas having different amounts or distribution of rainfall without making allowances for this difference in the basis of the results. The consumptive use from irrigation separately from the rainfall has been determined for some areas served by Kings River,[69] in California, the values derived being about 1.7 ft for orchards and vines and 2.0 to 2.25 for areas mainly in alfalfa. The mean rainfall is about 0.75 ft occurring during the winter months and should be added to these figures to give results in the terms used by the authors. Effective heat is available in all months of the year and some growth occurs throughout the year. Consumptive use by the trees and vines during their dormant period is replaced by the consumptive use of volunteer or planted cover crops, which receive their moisture supply mainly from the rainfall. The total annual effective heat for Fresno, Calif., is about 16,000 day-degrees for which Fig. 2 indicates a general consumptive use of 3.4 ft. This is a larger rate of consumptive use than the local records support and agrees with the indications of the authors' points 18 to 20 that consumptive use by crops will seldom exceed 3 ft.

C. S. Jarvis,[70] M. Am. Soc. C. E.[70a]—An unusual, although apparently logical, approach to a difficult problem has been used by the authors, and they have brought forth interesting as well as concordant data concerning water use on the various projects and regions dealt with. These data agree so closely with the fragmentary information heretofore available, and supplement the previous authoritative statements so satisfactorily as to call for a continuation of the research to cover many more river systems.

The writer's recent approach to consumptive use of water within a number of river basins was incidental to tracing the variations of annual or other periodic river discharge as related to recorded precipitation. It became apparent, for example, in dealing with the upper Missouri River and the entire

[69] "Ground Water Resources of Southern San Joaquin Valley," by S. T. Harding, *Bulletin No. 11*, California Dept. of Public Works, Div. of Eng. and Irrig., 1927, and later records.

[70] Hydr. Engr., SCS, Dept. of Agriculture, Washington, D. C.

[70a] Received by the Secretary August 12, 1941.

Platte River basins, that the yield from the headwaters and the precipitation were closely related for separate years, pairs or groups of years, such as decades, and up to the total periods of record, whereas the recorded discharges along the middle valleys showed more and more marked deficiency as the irrigated area, and the storage facilities serving such regions, became further developed. So persistently were these deficiencies displayed within the main irrigated regions that it became advisable to make an evaluation of such "increased consumptive use due to irrigation and storage projects." It thus appears that the quantities derived by the writer are not strictly comparable to those published by the authors, although there is some fairly close relationship. Any river basin in the arid region on which considerable regulation and utilization of its water resources have been achieved should show a marked reduction of outflow as a result of such development. For example, the Platte River discharging at Ashland, Nebr., can be reconciled with the summation of inflows from its four main tributaries only by allowing for some 1,600 cu ft per sec as representing consumptive use due to irrigation developments during recent years in the lower valley, as shown in Table 13.

Similarly, the Missouri River discharge at Fort Benton, Mont., averaging 4,608 cu ft per sec for the 5-yr period from 1934 to 1938, apparently should be increased by about 1,400 cu ft per sec to account for the increased consumptive use due to irrigation in upstream areas.

Utilizing the 1930 census data on "irrigation of agricultural lands," as shown in Table 14, and calculating the consumptive use for irrigated acreage in accordance with the quantities given in Tables 1 to 6, produce the results in the final columns of Table 14, some of which are but estimates, while others are official record data. It is obvious that the most nearly complete utilization of water resources among the larger drainage basins as of 1929 was along the Platte River, in Colorado and Nebraska, where increased consumptive use due to irrigation in the entire valley nearly equals the average outflow discharge at Ashland, within 25 miles of the Missouri River. However, if the Platte River discharge is measured near Duncan, Nebr., instead of at Ashland, the 10-yr average is 1,668 instead of 4,968 cu ft per sec, thus indicating that more nearly $\frac{4{,}780}{4{,}780 + 1{,}668} = \frac{4{,}780}{6{,}448} = 74.1\%$ of the available discharge is utilized within the region where irrigation is required, instead of approximately 50% of the total above Ashland including some areas where only supplemental if any irrigation is needed. Both the Great Basin and the Rio Grande drainage areas show utilization of water representing slightly more than 40%, computed as in the foregoing, for the year 1929, whereas the San Joaquin and Colorado River basins show 29.6% and 24.1%, respectively.

It is known that in many small drainage basins, components of the larger ones listed in Table 14, the utilization has closely approached 100% in recent years. However, where open storage over a long period is required to accomplish such development, considerable actual loss is necessarily entailed through evaporation, year after year, in addition to the inevitable deep percolation and seepage, small or otherwise in amount, which deplete the outflow without

adding materially to crop growth of such basins, thus reducing the net amount available for use.

Now consider the fact that the Mississippi River discharge at its mouth represents approximately 25% of the precipitation within the entire basin, with the Ohio River and smaller tributaries from the eastern portion yielding more nearly 50%, while the river systems draining the Great Plains area yield

TABLE 13.—MEAN DISCHARGE COMPARED WITH INCREASED CONSUMPTIVE USE FOR IRRIGATION ALONG THE PLATTE RIVER SYSTEM, COLORADO AND NEBRASKA

Item No.	Stream and station	Drainage area, in sq miles	Length of Record, in Years, to September 30, 1938	
			Observed	Estimated
1	South Platte River, South Platte, Colo.	2,050	36	....
2	South Platte River, Julesburg, Colo.	20,600	34	....
3	North Platte River above Pathfinder Dam, Wyoming	7,410	22	....
4	North Platte River below Pathfinder Dam, Wyoming	10,700	31	....
5	North Platte River, North Platte, Nebr.	32,000	43	....
6	Loup River, Columbus, Nebr.	14,200	24	....
7	Elkhorn River, Waterloo, Nebr.	6,390	12	24
8	Totals, items 2, 5, 6, and 7	73,190		
9	Ungaged intervening areas	10,610		
10	Consumptive use, storage, and losses in transit chargeable to irrigation	....	....	24
11	Platte River, Ashland, Nebr.	83,800	10	24

TABLE 13.—(*Continued*)

Item No.	Record-Period Average Discharge			Latest 10-Yr Mean			Latest 5-Yr Mean		
	Cu ft per sec	Cu ft per sec per sq mile	Depth per yr on drainage area, in inches	Cu ft per sec	Cu ft per sec per sq mile	Depth, in inches	Cu ft per sec	Cu ft per sec per sq mile	Depth, in inches
1	391	0.1907	2.59	323	0.1575	2.14	306	0.1494	2.03
2	485	0.0235	0.32	(300)	0.0146	0.20	201	0.0098	0.13
3	1,600	0.2159	2.93	1,303	0.1759	2.39	1,113	0.1502	2.04
4	1,784	0.1667	2.27	1,488	0.1391	1.89	1,197	0.1119	1.52
5	2,540	0.0794	1.08	(1,700)	0.0531	0.72	1,212	0.0379	0.47
6	2,984	0.2101	2.85	2,750	0.1937	2.63	2,574	0.1813	2.46
7	(776)	0.1214	1.65	711	0.1113	1.51	549	0.0859	1.17
8	6,785	0.0927	1.26	5,461	0.0746	1.01	4,536	0.0620	0.84
9	(984)	0.0927	1.26	(1,119)	0.1046	1.42	(1,082)	0.1920	1.39
10	(−1,469)	....	....	(−1,583)	....	....	(−1,679)	....	....
11	(6,300)	0.0752	1.02	4,988	0.0595	0.81	3,939	0.0470	0.64

more nearly 5% to measurable channel discharge and undetermined amounts to subsurface flow. Then the high utilization percentage of the Platte River (74.1 at Duncan or 48.9 at Ashland) applied to 5% runoff yield, results in only 3.7% or 2.5% of the precipitation which the consumptive use for irrigation diversions may represent, even though it largely depletes the Platte River discharge. The consumptive use due to both natural agencies and artificial

devices or practices represents more than 96% of the total rainfall, with less than half the optimum moisture requirements maintained throughout more than 90% of the drainage basin, and only about 10% of the area adequately supplied, the irrigated and high mountainous areas with plentiful moisture

TABLE 14.—IRRIGATED ACREAGE AND ESTIMATED CONSUMPTIVE USE OF WATER FOR IRRIGATION IN 1929

Drainage system	1930 census irrigated acreage of 1929	Estimated average consumptive use, in cu ft per sec	Record-period average discharge, in cu ft per sec [a]	Estimated utilization for irrigation; 1929 (%)
Missouri River	4,185,180	(9,650)	(75,000)	11.4
Jefferson River	363,380	(750)	1,541	32.7
Milk River	88,218	(182)	(500)	26.7
Big Horn River	361,926	(750)	2,211	25.3
Yellowstone River	861,145	(1,760)	9,365	15.8
Cheyenne and Belle Fourche rivers	71,550	(148)	(1,200)	11.0
Platte River	2,315,297	(4,780)	4,988	48.9
Kansas River	26,139	(640)	4,881	11.6
Republican River	23,387	(580)	777	42.7
Mississippi River, exclusive of Missouri River	902,560	(3,050)	530,000	0.6
Arkansas River	753,533	(2,560)	37,750	6.4
St. Francis River	13,282	(27)	1,424	1.9
White River	85,884	(176)	24,570	0.7
Red River	39,298	(130)	30,000	0.4
Gulf Streams other than Mississippi and Rio Grande	662,958	(2,370)	....	....
Rio Grande	1,468,913	(5,040)	7,350	40.7
Independent streams in Rio Grande Basin	95,812	(400)	....	....
Colorado River	2,537,124	(7,250)	22,800	24.1
Great Basin drainage	2,069,986	(5,280)	(7,000)	43.0
Columbia River	3,393,640	(7,020)	(300,000)	2.3
Snake River	2,339,264	(4,840)	48,860	9.0
Independent streams of Snake River Basin	82,074	(170)	....	....
Pacific Ocean streams other than Columbia	4,225,971	(10,750)	(75,000)	12.5
San Joaquin	2,405,380	(6,970)	16,550	29.6
Western States	19,547,544	(56,000)	(1,050,000)	5.1

[a] Unofficial estimates or derivations are in parentheses.

being of nearly equal extent, or about 5% of the basin area in each classification. Furthermore, it is evident that the limit of practicable irrigation development within the Platte River basin is within view, unless importations of additional volumes are to be accomplished.

R. W. DAVENPORT,[71] M. AM. SOC. C. E., AND GORDON R. WILLIAMS,[72] ASSOC. M. AM. SOC. C. E.[72a]—In the period from 1935 to 1939, the writers participated in studies,[73] made by the U. S. Geological Survey, which produced results that may be compared with those presented in the paper. The studies consisted of the determination of natural water loss (or consumptive use) for about 200 non-irrigated drainage areas in humid and sub-humid regions east

[71] Principal Hydr. Engr., U. S. Geological Survey, Washington, D. C.

[72] Hydr. Engr., U. S. Engr. Office, Baltimore, Md.

[72a] Received by the Secretary August 20, 1941.

[73] "Natural Water Loss in Selected Drainage Basins," by Gordon R. Williams and others, *Water-Supply Paper No. 846*, U. S. Geological Survey, 1940.

of the Rocky Mountains. The areas studied are similar to areas 10 to 20, in Table 1.

The relation between temperature and consumptive use was determined for only twenty-eight areas and in the published report[73] mean annual temperatures were used instead of day-degrees. It was realized that day-degrees of mean daily temperature above 32° might give a better correlation than mean temperatures. In order to determine the relation between mean temperature and day-degrees above 32° for yearly periods, the records for twenty-three representative stations, totaling sixty-nine record years, were studied. The results of the study were plotted and a mean curve drawn.[74] This curve showed that, for mean annual temperatures above 48° F, the mean temperature should give as good a correlation as total day-degrees.

Beyond showing the relation between the two methods of indicating temperature no use was made of this curve but it could be used to change the mean annual temperature to equivalent total day-degrees above 32° F. This has been done for the purposes of this discussion and the results are shown in Fig. 5. The plotted points in Fig. 5 are identified by their numbers as follows:

Point No. (Fig. 5)	Station
1	South Branch Nashua River at Clinton, Mass.
2	Sudbury River at Framingham Center, Mass.
3	Lake Cochituate Outlet at Cochituate, Mass.
4	West River at Newfane, Vt.
5	Swift River at West Ware, Mass.
6	Middle Branch of Westfield River at Goss Heights, Mass.
7	Clearfield Creek at Dimeling, Pa.
8	Swatara Creek at Harper Tavern, Pa.
9	Upper Little Swatara Creek at Pine Grove, Pa.
10	Oconee River near Greensboro, Ga.
11	Chattahoochee River near Norcross, Ga.
12	Conecuh River near Andalusia, Ala.
13	East Fork of Tombigbee River near Fulton, Miss.
14	Pearl River at Edinburg, Miss.
15	Red Bank Creek at St. Charles, Pa.
16	Miami River at Dayton, Ohio
17	West Fork of White River at Noblesville, Ind.
18	Tittabawassee River at Freeland, Mich.
19	Red River at Fargo, N. Dak.
20	Red River at Grand Forks, N. Dak.
21	La Crosse River near West Salem, Wis.
22	Kickapoo River at Gays Mills, Wis.
23	Blackwater River at Blue Lick, Mo.
24	South Grand River near Brownington, Mo.
25	Little Arkansas River at Valley Center, Kans.
26	Walnut River at Winfield, Kans.
27	Neches River near Rockland, Tex.
28	Angelina River near Lufkin, Tex.

74 "Natural Water Loss in Selected Drainage Basins," by Gordon R. Williams and others, *Water-Supply Paper No. 846*, U. S. Geological Survey, 1940, Fig. 4.

The consumptive use was originally computed in inches but has been changed to feet to be comparable with the authors' study.

The differences in procedure between the two studies are listed in Table 15. The greatest difference lies in the method of determining effective heat. If the authors' curve from Fig. 2 is drawn on Fig. 5, a close similarity is shown in

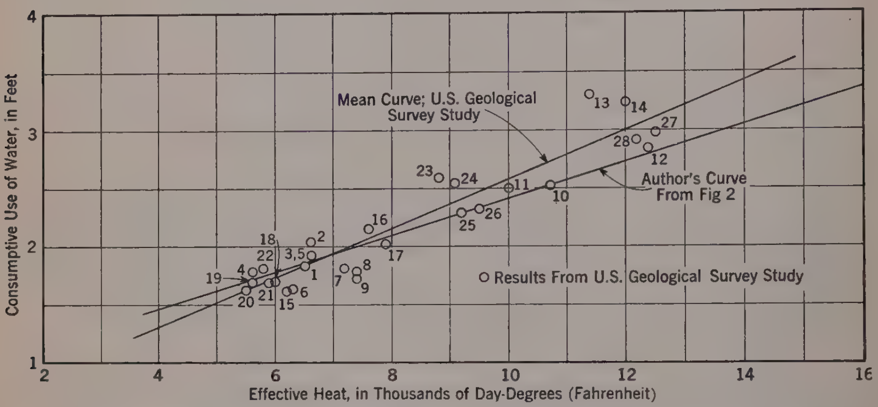

FIG. 5.—COMPARISON OF RELATIONS BETWEEN CONSUMPTIVE USE AND EFFECTIVE HEAT, USING DIFFERENT METHODS OF COMPUTING EFFECTIVE HEAT

the slopes and positions of the curves even though the bases for computing the total day-degrees are different. The conclusion to be drawn is that the two methods of computing effective heat have compensating influences, that they lead to similar totals in day-degrees and are perhaps equally effective methods of correlating consumptive use to climatic conditions.

TABLE 15.—COMPARISON OF METHODS

Item	Authors' study	Geological Survey study[a]
Period used in computing consumptive use	Calendar-year	Water year ending Spetember 30
Correction for years of short water supply	Corrections made	No corrections
Period used in computing effective heat	Growing season	Water year ending September 30
Temperature ranges used in computing effective heat	Degrees by which maximum temperature for the day exceeds 32° F	Degrees by which mean temperature for the day exceeds 32° F

[a] "Natural Water Loss n Selected Drainage Basins," by Gordon R. Williams and others, *Water-Supply Paper No. 846*, U. S. Geological Survey, 1940.

There are notable differences between the irrigated valleys and non-irrigated watersheds studied, shown in respect to altitude and various climatic features. Moreover, the way in which the water supply comes to the land is different, that for the irrigated valleys coming to a greater degree in a time when the capacity for use is greater. In view of these differences the correspondence of the relation between consumptive use and effective heat for the two kinds of areas is especially interesting and suggests that here also compensating influences are in operation.

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DISCUSSIONS

THE SURVEYOR AND THE LAW

Discussion

By Messrs. E. F. Chandler, Harry Rubey, William H. Richards, Jr., and C. B. Humphrey

E. F. Chandler,[61] M. Am. Soc. C. E.[61a]—Special commendation is due this paper because it brings so definitely to attention one very important feature of the work of a land surveyor making field measurements to determine the proper location of doubtful or disputed boundaries. This is the fact that if the surveyor is not merely careful in his surveys but also fairly informed on a certian line of legal questions he is often able (unofficially, but by willing consent of both parties) really to fulfil a judicial function and bring about fairly harmonious agreement between parties who might instead have resorted to costly hearings in court.

The nontechnical purchaser is quite likely to think, and can easily be misled to believe, that his deed gives absolute and complete legal title to the lengths and acreages specified by it, which of course is not invariably true.

For example, careless work in the early surveys many years ago may have vitiated all the data recorded in them. A striking case of this came before the writer in his own early experience, in south central Wisconsin, where there was a dispute concerning boundaries between adjoining land owners, each of whom had a deed of very convincing prima facie appearance. In both cases, dimensions were taken directly from the plats of the original government contract surveyor, who had staked off the townships and sections there in some year of the late 1840's. About fifty years later, the writer retraced a series of the lines, and was fortunate enough still to find indisputable evidence (witness trees, bearing trees, etc.) at almost every adjoining section corner and quarter-section corner. On one line between corners (east and west in Section 6, and therefore not assumed or reported as a precise mile) a so-called mile was found to be actually about 900 ft less than the figures certified for it by the original contract surveyor. No surplus was in any adjoining section that could by any possibility be obtained on the plea of an alleged mistake or blunder by the original surveyor in accidentally witnessing some trees at an unintended point. In this case, however, the land had been so long occupied without objection that it was easy to make the most injured claimant realize that,

Note.—This paper by A. H. Holt, M. Am. Soc. C. E., was published in May, 1941, *Proceedings*.

[61] Prof., Civ. Eng. and Dean Emeritus, College of Eng., Univ. of North Dakota, Grand Forks, N. Dak.

[61a] Received by the Secretary July 7, 1941.

although his quarter section was really 900 ft narrower than the distances alleged in his deed, it would be a useless expense to bring any suit or to attempt to reclaim something that actually did not exist.

To avoid stirring up needless claims or disputes based upon present-day failures of tracts to fit precisely the reported metes and bounds certified by the surveyors of earlier days (some of whom were quite hurried, careless, and inaccurate, and some even worse in framing up semi-fraudulent description), the land surveyor now should keep clearly in mind the usual rule for accepting priority of calls in the early surveys, which holds (except in special or unusual cases where there is extra evidence supporting one or another of the conflicting items)—namely, the general rule that definite unchanged landmarks, monuments, or permanent stakes and fences outweigh all other evidence; lengths and distances have less weight, but outweigh directions or bearing; bearings are good evidence only as compared with areas; and areas have little or no value in fixing boundary-line locations, being merely corroborative unless the other calls are lacking.

The author refers briefly to another principle that the surveyor must constantly remember if he does not wish to encourage his client in useless disputes—the doctrine of "adverse occupancy." Briefly worded, in most states undisputed continuous possession "under color of title" through ten years gives the occupant indisputable ownership; or even without color of title he gains ownership in twenty years. In different states there are variations, so that in some localities even half this time (five years, or ten years) is sufficient; there are different technical exceptions in different places (such, perhaps, as the law not beginning to hold against minor owners until they reach the age of twenty-one, and never holding against the general public, but merely against individual owners, etc.); and, of course, the occupancy must have been absolutely undisputed, without legal objection at any time in the entire period. The surveyor must, perforce, keep in mind the effect of that statute of limitations, whatever its precise form may be in that jurisdiction, and must not unduly encourage his client to hope to gain or regain, merely by mathematical measurements, the possession of real estate, the ownership of which has really been lost by years of neglect.

If the surveyor is careful, however, is fairly accurate in the engineering work of his surveys, has a well-founded general idea of the legal principles attaching to them, and has good judgment in avoiding the appearance of prejudice or bias, he will usually find his recommendations accepted (perhaps rather unwillingly, but because seeming the most reasonable to both sides) without any need for final resort to courts of law, or to the vote of a jury of twelve men, none of whom is likely to have more than the merest superficial knowledge of such matters.

If a case does actually come to court trial, the evidence of a surveyor as an expert witness on matters to which he can properly testify should be practically incontrovertible. Furthermore, as stated by the author, if he does not claim greater proficiency in legal knowledge than the lawyer himself, the lawyer probably will be glad of his advice or assistance on the technical sides of the question. If the case is quite important, the lawyer may even (as has repeatedly befallen the writer) ask the surveyor-engineer to make for him the preliminary draft of the engineering part of his brief.

HARRY RUBEY,[62] M. AM. SOC. C. E.[62a]—This is a valuable paper by the author on the professional aspects of a subject too little understood by engineers. The Society is indeed indebted for the many references that accompany the paper—references that most engineers would otherwise overlook. From the wide range of possible discussion of the paper, the writer will attempt only to emphasize the professional nature of the work, as distinguished from the subprofessional status ordinarily assigned to it.

Too often the subject of boundary surveys is discussed and practiced as a distinctly subprofessional activity. The author is entirely correct in implying that a highly professional engineering judgment, often involving a considerable legal knowledge, is required for boundary surveys that fall outside of a simple routine.

The actual surveying—that is, the making of measurements and calculations—frequently might be classified as subprofessional; but when the engineer so engaged is called upon to exercise legal knowledge and judgment in connection with surveys, the work takes on a professional character, and in many cases may become highly professional.

Although the private engineer possesses no formal legal status other than that of expert witness, actually and informally he must advise his client and conduct his surveys in accordance with accepted legal principles. In many cases, the engineer must cooperate on a definitely professional basis with an attorney, both in the preparation of a case and in the negotiations or court proceedings connected therewith. From the moment the engineer is approached by a client regarding a boundary survey, his advice and action must be based on sound legal knowledge and professional judgment.

When it is realized that the interests of the client, the engineer, and the community are best served by securing the facts, by studying the case, by definite solution from a survey, by settling the controversy through agreement or by arbitration, and perhaps finally by court action, the professional nature of the engagement becomes apparent.

Another definitely professional phase of this subject in which the Society cooperates with the American Bar Association was stressed in 1941.[63]

It would be most valuable if the author could give in some detail the engineer's ethics, techniques, and procedures in the matters of arbitration and expert testimony.

All too few qualified engineers and attorneys are available for professional boundary survey assignments, and the published information and discussion of the matter are entirely too limited.

WILLIAM H. RICHARDS, JR.,[64] M. AM. SOC. C. E.[64a]—Professor Holt renders a service to those interested in boundary surveys in his discussion of the duties and responsibilities of surveyors when engaged in recovering and establishing

[62] Chairman, Civ. Eng. Dept., Univ. of Missouri, Columbia, Mo.

[62a] Received by the Secretary August 4, 1941.

[63] "Land Surveys and Titles," Second Progress Rept. of the Joint Committee of the Real Property Div., American Bar Association, and the Surveying and Mapping Div., Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., June, 1941, p. 1065.

[64] Cadastral Engr., Gen. Land Office, Div. of Surveys, Washington, D. C.

[64a] Received by the Secretary August 4, 1941.

boundaries of land. His paper indicates the combined technical and legal character of the land surveyor's work and demonstrates the necessity for some knowledge of the law on this subject and of legal procedure by the civil engineer who engages in this field of activity.

Under the heading "Definition and Ownership of Field Notes" the author observes that field notes, although defined in Corpus Juris as "Notes made by the surveyor in the field while making the survey," also, apparently, under some usages, refer to a description subsequently written up from the notes actually taken in the field. He mentions in this connection the "original field notes" of the public land surveys.

There is a very definite distinction between the notes and data taken by the private surveyor during the progress of his field work and the official field notes of the public land surveys. The current edition of "The Manual of Instructions for the Survey of the Public Lands of the United States" contains the following statement in this connection: "Section 542 The initial notes are kept in pocket field tablets. The final field notes for filing are transcribed from the field tablets, and are typewritten upon regulation field note paper."

The 1881 edition of the manual of instructions contained the following statement relative to field notes:

> "From the data thus recorded [in field tablets] at the time when the work is done on the ground, the deputy [surveyor] must prepare true field notes of the surveys executed by him, in the manner hereinafter prescribed, and return same to the surveyor general, together with the required sketches, at the earliest practicable date after completion of his work in the field."

Such final or true field notes, when duly certified by the surveyor, approved by the supervisor of surveys (formerly by surveyor general), and accepted by the commissioner of the General Land Office, are the official field notes.

The actual running of the lines of a survey in the field and the recording of the notes are not alone sufficient to constitute an official survey. Until all the requirements as to approval are complied with, the survey and its record are not regarded as complete, and the lands are still to be classed as unsurveyed. In other words, to justify the application of the term "surveyed" to a body of public land, something is required beyond the completion of the field work—namely, approval by the authorized officials of the federal government.[65]

C. B. Humphrey,[66] Esq.[66a]—It is well that, from time to time, there appear, in the technical publications that reach the land surveyor, papers dealing with the problems of the surveyor in establishing and re-establishing boundary lines, and to remind him that, although the true lines of ownership are matters for a court, he may report the lines to the best of his knowledge.

Professor Holt's paper dealing with this subject is timely, when communities in the United States, with Federal Aid, are establishing coordinate systems to

[65] See U. S. Supreme Court decision in the case of Cox vs. Hart, 260 U. S. 427.

[66] Chf. Engr., Massachusetts Land Court, Boston, Mass.

[66a] Received by the Secretary September 2, 1941.

which may be referred all public and private surveys. His remarks upon the duties and limitations of a surveyor are most instructive, especially to young surveyors who, at times, seem to overlook the fact that most of their problems, pertaining to the interpretation of a boundary line, already have been determined by a court and the data acquired by a little research work.

The writer recalls reading "The Judicial Functions of Surveyors," by Mr. Justice Cooley of the Michigan Supreme Court and joins with Professor Holt in recommending that it be read by all interested in survey work. It appeared as "Appendix A" in "Theory and Practice of Surveying"[67] in 1892 and the writer, in his official duties, often quotes parts of this paper.

With reference to the establishment of a disputed boundary line by parol evidence or by agreement of interested parties, the surveyor may perform his functions as a surveyor by making a survey, setting bounds and preparing a plan for recording. As Professor Holt states, however, it may not be a judicial determination (which is a matter for a court) but it may be adjusted by direct grants between the respective owners.

In a Massachusetts case[68] that was an appeal from a Land Court decision, there was evidence that the disputed line had been fixed by a parol agreement by predecessors in title and the court ruled that although parties could agree orally, no title would pass by force of such agreement unless followed by occupation.

In proceedings in the Massachusetts Land Court there are many cases in which, after all the evidence is presented, disputed boundary lines are settled by written agreement or stipulation between the parties. The line thus agreed upon becomes a part of the final decree of the court confirming the title to the land and fixing all boundary lines.

The author has carefully emphasized that the determination of a disputed boundary line by either of the foregoing methods does not necessarily mean that it is the exact location as laid down by the original surveyor and shown on some recorded plat referred to, and from which one could, with safety, measure to reproduce all other lines.

In a Massachusetts case[69] the court held that the decision in a previous case whereby the line of a street was determined, although subsequently it was found that it was not determined in accordance with the plan from which the petitioners obtained title, did not prevent the petitioner in an adjacent case from insisting upon having the line of the street located in its proper position where it bounded upon his land.

To whom belong the field notes for a private survey? The writer contends that they belong to the professional surveyor, but that the surveyor should deliver to his client a plan and report that would be of value to a subsequent owner, and that he should show what an expert surveyor would call proper engineering data, and not a plat with plotting lines drawn to scale with distances only.

He should further recommend, as Professor Holt has so well stated, that if

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[67] "Theory and Practice of Surveying," by J. B. Johnson, John Wiley & Sons, New York, N. Y., 1892.
[68] Crawford vs. Roloson, 256 Mass. 331.
[69] Pollard vs. Burchard, 199 Mass. 376.

no monuments exist, sufficient monuments should be set to perpetuate and mark the lines surveyed and established. It is not always the fault of the surveyor that monuments are not set, but due to a short-sighted policy on the part of his client, which, in many cases, is the cause of the plan and survey being practically worthless when retraced after many years.

The author has presented an unusual paper, with helpful references to court cases, and it is hoped that it will be read by all those engaged in surveying practise. The writer also wishes to call attention to the coordinate systems now established by many states, a part of the expense of which is being borne by the federal government. The use of these systems by the engineering departments of various political subdivisions will enable more points to be established and should to a certain extent eliminate the duplication of survey work.

Many years have passed since G. Washington, Surveyor, placed the "corner stone" for the plantation under the Natural Bridge in Virginia and wrote his initials on the side of the cliff. The poplar trees, black oaks, white oaks and hickory corners mentioned in the "King's Grant" have long disappeared. Directions are no longer given in degrees only and distances in rods. The present-day surveyor, with modern instruments, must regard such common sources of errors as plumbing, alinement, temperature, wind, etc., and must think in terms of coordinates when locating a point on the earth's surface. He must also have in mind the standard used by previous engineers in laying down the Geodetic Control Stations if he is to agree with subdivisions between control lines. With these precautions, and coordinate values of record on recorded plats, there should be fewer disputes in the future as to the location of boundary lines.

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DISCUSSIONS

COMPACTION OF COHESIONLESS FOUNDATION SOILS BY EXPLOSIVES

Discussion

By Reuben M. Haines, Assoc. M. Am. Soc. C. E.

Reuben M. Haines,[4] Assoc. M. Am. Soc. C. E.[4a]—The initial tests and first actual application of the method of compacting loose, cohesionless foundation soils by detonation of buried charges of explosives are described in this paper. Colonel Lyman is to be highly commended for conducting the investigations, and for his presentation of data, since this method of compaction may provide an economical and practical solution to produce a suitable foundation for many structures. Before the limitations of the method are known, additional information and data will need to be obtained by future field and laboratory investigations. Since the author has proved that the method is practicable for certain conditions, it is believed that additional information will become available in the near future. The costs of making preliminary investigations where the method might be used would appear to be inexpensive. Such investigations should be made to determine if settlement can be obtained for the particular soil deposit and to determine the spacing, order, and size of charges that would give the most compaction. It appears that the further development of the method is highly desirable. For certain soil conditions, it may be possible that the use of well points to remove excess water after detonation, or the loading of the area, will produce greater compaction.

This method of compaction can be used only for soil deposits in which a new and denser soil structure will be formed if the material is completely disturbed. The compaction that can be obtained is limited and depends upon the type of soil particles, type of soil structure, the initial density, the permeability, the drainage conditions, the stress conditions, and the degree of saturation. The detonation of the buried charges in a saturated loose deposit produces a liquid or partly liquid mass by destroying the soil structure. The particles of soil settle to form a new structure as the excess moisture seeps from the mass. The rate and quantity of seepage probably affect the degree of

Note.—This paper by A. K. B. Lyman, M. Am. Soc. C. E., was published in May, 1941, *Proceedings*.

[4] Engr., Corps of Engrs., U. S. Army, U. S. Engr. Office, Massena, N. Y.

[4a] Received by the Secretary September 17, 1941.

compaction obtained after the detonation of a charge. After the excess moisture has escaped and the new soil structure formed, the detonation of another charge produces additional compaction; however, the data of the tests at Denison Dam show that there will be less change in density than by the detonation of the previous charge. The number of coverages to produce optimum compaction can be determined only by field tests. It would be interesting if Colonel Lyman, in the closing discussion, would present any additional available data that may indicate the change in density by the detonation of each successive charge. The writer believes that the inertia and tamping actions produced at the time of detonation have little effect upon the compaction of saturated materials; however, if the soil deposit is only partly saturated, these actions may be primary causes of the compaction of the soil particles as rearranged by resettlement. The writer believes that only a minor degree of compaction can be obtained by this method unless there is sufficient moisture to produce partial liquefaction when the soil structure is shattered.

The development of the method may lead to many practicable uses. During the construction of many earth dams, the current procedure is to dewater the channel areas and place rolled-fill materials. Investigations may develop this method so that cohesionless material may be placed and satisfactorily compacted in the channel areas of some projects without the construction of cofferdams and dewatering of the area. Until more is known about the stress-strain relations of cohesionless materials when subjected to disturbances, and until the critical void ratios for various stress conditions can be determined, it will not be known if this method can be used to produce a density below the critical void ratio for any imposed loads of appreciable magnitude. Any foundation, however, would have a satisfactory density to insure against liquefaction if this method is used after the stresses of the imposed load are produced in the deposit; but this procedure may be very expensive for large earth-dam projects. The method may be economical and practicable for many earth-dam projects, as in the case of the Franklin Falls Dam where only a moderate degree of compaction of the foundation deposits on the terraces was required due to foundation and design features.

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DISCUSSIONS

EXTENSOMETER STRESS MEASUREMENTS, NORTH AVENUE BRIDGE, CHICAGO, ILL.

Discussion

BY THEODORE BELZNER, AFFILIATE AM. SOC. C. E.

THEODORE BELZNER,[15] AFFILIATE AM. SOC. C. E.[15a]—The statement made by the authors that few measurements of stresses on structures in service had been taken until about 1917 is incorrect. Many strain-gage measurements in actual construction or maintenance work on bridges prior to that period had been made. The writer firmly believes that a complete bibliography on this subject should be brought substantially up to date.

One case in particular, in a bridge of large span, is the Williamsburg Bridge across the East River in New York, N. Y., not given in the Appendix. Prior to 1917, stresses were measured on the Williamsburg Bridge during the summer of 1911; they were measured under train movements; also, during 1913 and 1914, when a comprehensive series of extensometer investigations were made in connection with the strengthening of the end spans of this bridge. This work was done[16] by the Department of Bridges (later the Department of Plant and Structures, and finally Department of Public Works), City of New York, in cooperation with the National Bureau of Standards, under the auspices of the late James E. Howard, engineer physicist, on the important truss members at the main towers and legs of the Brooklyn intermediate towers.

The precision attained with the Howard extensometer used in this test proved to be an invaluable aid, and its merits were illustrated in certain vital operations, especially with the various stages of wedging, and in the transferring of stresses from the old to the new diagonal members of the end trusses.

The writer agrees with the authors that field stress measurements "require a high order of instrumental and technical skill in order to be of value." The skill required to obtain reliable results in strain-gage measurements is not in

NOTE.—This paper by Lawrence T. Smith, M. Am. Soc. C. E., and Paul Lillard, Esq., was published in May, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1941, by Messrs. Arthur R. Lord, and J. Charles Rathbun.

[15] Insp. of Steel and Bridge Insp. in Chg., Brooklyn Bridge, Dept. of Public Works, City of New York, Brooklyn, N. Y.

[15a] Received by the Secretary June 23, 1941.

[16] Discussion by Isidore Delson of "Stress Measurements on the Hell Gate Arch Bridge," by D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. LXXXII, December, 1918, pp. 1087–1089.

the possession of all. In other words, the most important objective to be considered in taking stress-strain measurements (other factors being equal) is to obtain consistent results; and, in order to secure such results (which are absolutely essential), persistent patience and painstaking thoroughness, combined with good judgment on the part of the observer, are required, until he experiences little or no difficulty. It is upon the observer that the value of such measurements will depend.

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DISCUSSIONS

TRAFFIC ENGINEERING AS APPLIED TO RURAL HIGHWAYS

Discussion

BY STEPHEN E. BUTTERFIELD, ASSOC. M. AM. SOC. C. E.

STEPHEN E. BUTTERFIELD,[6] ASSOC. M. AM. SOC. C. E.[6a]—The use of certain corrective engineering techniques to aid in the modernization of existing highways is a practice which to date has been used by a limited number of the more progressive state highway departments. From the point of view of more efficient highway usage, either in terms of accident reduction or increased roadway and intersection capacity, this practice is one that might be used to advantage to a far greater extent. Because many state highway departments are now (1941) faced with curtailed operating revenues, due either to the diversion of gasoline tax moneys or to national defense restrictions in the sale of gasoline, many thousands of miles of antiquated highways must be made to serve as best they may for some years to come. With this the case, it will be necessary for many states to revamp certain of their older highways by the use of any or all of the corrective techniques known to traffic engineers.

As clearly outlined by Mr. Harris, one of the first steps in such a program is the detection of the points of greatest hazard by means of accident records. With the application of the correct treatment to each point of hazard, a sound approach is afforded for a program of selective remedial measures. In applying this technique to several specific examples, the author has indicated how gratifying may be the results of such a selective procedure.

Closely akin to those points of hazard that are revealed by accident records are other points of secondary hazard which are not as easily recognized. They result in congestion and frequently cause accidents, but they do not always appear in the form of such familiar bottlenecks as narrow pavements, bridges, or underpasses. However, under conditions of fairly heavy volumes of traffic, they soon become evident. Frequently these bottlenecks may be intersections of types which are unable to adequately handle even the average volumes of

NOTE.—This paper by Milton Harris, Assoc. M. Am. Soc. C. E., was published in May, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1941, by Charles E. Conover, M. Am. Soc. C. E.

[6] Traffic Engr., State Highway Dept., Hartford, Conn.

[6a] Received by the Secretary September 15, 1941.

traffic which pour into them for interchange. Thus it is that the capacity (and, therefore, the efficient use) of a roadway may depend almost exclusively upon the frequency and capacity of its interchanges. If this is the case, the solution obviously lies in remodeling the offending intersections. Progressive remedial steps are: The construction of additional turning lanes, channelizing islands, traffic signals, or clover-leaf grade separations.

An invaluable tool in gaging the efficiency of a highway system is the "speed and delay study." Mr. Harris has touched on this subject in his consideration of over-all speeds in Case No. 3. From a study of the speed profiles the points of congestion can be located readily and accurately. By the use of these data the proper corrective treatment can be applied to those specific locations, and only those locations, which are at fault.

The author states that in applying traffic engineering to rural highways the most outstanding feature that needs correction is the present structure, which must be made safe for existing traffic. Unfortunately, some state highway engineers, lacking the essential basic facts concerning points of congestion and locations of accidents, have felt that there is little or no salvage in an older roadway structure. Therefore, depending upon the funds available, a particular stretch of highway that is known to be unsatisfactory for economic transportation is either completely rebuilt or postponed for possible inclusion in the reconstruction program at some later date. Would it not be a wiser procedure, if only limited funds are available, to modernize the old highway in such a way that it may serve fairly efficiently for the additional period? This cost might be almost insignificant compared to the cost of complete reconstruction.

As stated by Mr. Harris, the full use of all available traffic data is essential in order to design a highway that functionally fulfils its sole purpose of providing efficient transportation. The successful highway designer must not only interpret traffic data in terms of present conditions, but he must look into the future. He must boldly appraise and carefully estimate the demands that traffic will make on the proposed highway twenty or even thrity years hence. The design of a highway for current operating highway speeds may result in the construction of a highway already antiquated at the time of its completion.

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DISCUSSIONS

PILE-DRIVING FORMULAS

PROGRESS REPORT OF THE COMMITTEE ON THE BEARING VALUE OF PILE FOUNDATIONS

Discussion

By Messrs. Robert D. Chellis, Lazarus White, John G. Mason, Carlton S. Proctor, George Paaswell, and Abraham Woolf

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Robert D. Chellis,[19] M. Am. Soc. C. E.[19a]—The Committee is quite right in laying emphasis on the fact that the subsoil and foundations are an integral part of the design of the structure. Considering how minutely the stresses in each beam, column, and rivet are computed, and comparing this expended effort with the vague procedure often followed in the soil until the foundation concrete level is reached, it is evident that a sad disparity exists.

There is much more involved in designing and driving piles than the simple application of a mere formula, which is one tool among others, and the comments in this discussion will emphasize many of these considerations which were not mentioned in the Report, but which are equally necessary to an intelligent handling and solution of the problem.

Before the matter of a pile-driving formula is reached, however, full data as to the subsoil should be available for consideration. Borings in adequate number are generally most important, but are often not obtained. It may be very dangerous to rely on hearsay, or data from similar or nearby sites. The writer has found many cases in which the soil no farther away than across the street was of entirely different composition and depth. Even a few feet away, great differences have been observed. These facts indicate the need of a sufficient number of borings.

Piles should be driven for four reasons—namely, to compact the soil in order to increase the soil value; to transfer the load down to a stratum considered more capable of carrying the load than the upper strata; to act as columns to transmit the load through softer strata to end bearing on rock or

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Note.—This Report was published in May, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1941, by Messrs. G. G. Greulich, C. O. Emerson and D. O. Northrup, Harry J. Engel, and John D. Watson.

[19] Engr., Structural Div., Stone & Webster Eng. Corp., Boston, Mass.

[19a] Received by the Secretary August 15, 1941.

hardpan; or to provide uplift resistance against hydrostatic uplift or due to overturning action. In the past many cases have been observed in which piles were driven into clays of no better load-carrying capacity, or not as good, as the top strata, and in such cases the piles may have even been detrimental, due to disturbing the soil.

A study of the theory, test results, and driving results from various types and lengths of piles driven with different kinds of hammers over several years has shown the writer that the "Engineering News" formula (Eq. 1) is not a general answer to the problem. If its derivation and limitations (principally due to omission of terms) are understood, and the factor of safety assumed is realized, there are cases in which it is suitable for use. However, if the question is studied sufficiently to know these things, it is most likely that a more nearly correct procedure will be followed as standard practice in all cases. The catchword "simplicity" has been greatly overworked, as an argument for avoiding warranted study or effort in understanding the theory of pile-driving formulas. When reflecting on the enormous amount of computation and thought put in designing the superstructure of many structures, to use a formula of the Hiley type (Eq. 4), applied probably only once for the project, appears a trifling item; and, in fact, it is trifling. There should be no reason why the engineer responsible for the entire design should desire a formula that can be handled by a pile inspector, any more than he should desire formulas by which the concrete inspector and ironworkers could design the floors and columns, unless it is possibly an effort to avoid the issue due to lack of understanding of the proper method of handling the design of piles. The principal need in applying the Hiley formula is to understand the terms, and to have available the data and coefficients for substitution in the formula. The writer has used this formula since 1935 on a great variety of projects and has found its use very practicable.

The Report indicates that the Committee hopes, by means of reported test data, to evolve a formula for general use. It would seem logical to approach the matter in two steps, the first of which is of as great importance as the second. This first step is to obtain uniformity in results when driving any kind of pile with any type of hammer. This will provide assurance that the final construction is not dependent on the types of piles and hammers used. The ground has no intelligent entity, and cannot be expected to know what kind of pile is in it, and since the purpose of a pile is to transfer a load from one elevation down to a lower one, it makes no difference what the material of the pile is, so far as the carrying capacity of the ground is concerned. Therefore, again, as far as the ground enters into the picture, the pile can be hollow, solid concrete, wood, or any other type. Furthermore, once the pile is in the ground to the same tip elevation, it makes no difference to the ground how it was driven there, as long as it is there. In certain conditions the matter of the amount of earth displacement is a factor, and should be considered when selecting the type of pile. The older formulas give widely varying results with different types of piles and hammers, entirely out of reason. The writer has found the factor of safety provided by the "Engineering News" formula, when investigated by the Hiley formula, to be as low as 0.5 and as high as 16.

No doubt even more serious variations may occur in other cases. Therefore, the first step might well be to obtain uniformity, and the second to set the level upon which to place this result obtained, or, in other words, the factor of safety. In this way cost comparisons can be made, considering various types of piles and equipment available, with the knowledge that the factor of safety is not varying appreciably; in other words, that the purchaser is getting the same engineering design results in each case. It is also very likely that once comparable results are obtained, the factor of safety can be set within reasonably correct limits, by means of judgment, and comparison of computed results with test data already available, and which it is most desirable to accumulate in accordance with the Committee's suggestion.

A-1. Definition.—The most useful results can be obtained by use of the dynamic formula, followed as a check by a consideration of the static resistance required to carry the pile load into the soil by friction and end bearing, except in cases of pure end bearing where the piles serve only as columns.

A-2. Limits of Formulas.—The derivation, assumptions, and limits of the pile formula used should most certainly be understood by the user. For this reason, there is advantage in using the full Hiley formula without simplification by omission of any terms. Little effort is required to retain all terms. Certain limitations, even to a pile formula that considers energy losses due to all sources (which are often disregarded through lack of knowledge), are as follows: Nonvalidity of a dynamic formula when driving in cohesive soils; lack of relationship between an individual pile value and the value of a group of piles, or soil capacity under the structure as a whole; reduction in useful driving energy at tip of pile, extending to complete disappearance of net driving energy after deducting losses, due to allowing double-acting hammers to operate under reduced steam pressure; reduced driving energy during underwater driving; reduced energy when driving batter piles; efficiency of hammer (see comments under efficiency of hammer). Since a large proportion of the applied energy is often consumed in impact and in elastic losses in the cap, pile, and quake of the ground, the formula is very sensitive at small penetrations and an appreciable decrease in applied energy may result in no driving energy reaching the tip.

A-3. Effect of Soil Types.—A dynamic formula should be used only when driving in cohesionless, or nearly cohesionless, soils for carrying capacity. Of course, frequently the pile will pass through cohesive strata on the way down to the carrying stratum. In some cases the pile may pass through a cohesionless upper stratum which it is not considered desirable to use for bearing, possibly because of the depth and soft consistency of a stratum below it, and be driven into a firm cohesionless lower stratum. In such cases the friction value due to the upper stratum should be deducted from the computed driving resistance. In some instances such an upper stratum may settle due to compression of the intermediate soft stratum, allowing the top stratum to drag down on the pile and adding to its load, in which event the allowable pile-carrying capacity may need to be reduced below that shown by a dynamic formula.

A-4. General Formula.—The nomenclature does not state the unit in which h is to be measured. In Hiley's formula (Eq. 4) it should be used in inches, whereas in the "Engineering News" formula (Eq. 6) and in the suggested Eqs. 8 and 9, it is in feet. This should be clarified. For the comments in this discussion h is taken in feet.

The term P is stated as the weight of the pile. This term should also include the weight of the driving cap, which, in the case of some piles such as light steel shells, may exceed the weight of the pile itself, thus, by neglect of this factor, invalidating the computations. In case a follower is used, this term should also include the weight of the follower.

A table of weights of caps, for the use of the engineer in making preliminary computations, is desirable. This information is not available in manufacturers' catalogs. The writer has published such a table[20] which, while incomplete, will serve as a guide. When the actual cap is available, the weight and details should be considered.

The term L is stated to be the length of the pile, which should be taken as the distance from the head of the pile to the center of resistance to driving, as discussed hereinafter. This value should be taken in inches, for use in Eqs. 10 and 11.

The values of e given in the Report are 0.75 for the usual drop hammer and 0.90 for a single-acting steam hammer. The writer has considered the values of efficiencies of various hammers under varying conditions, and suggests the values in Table 6, which have been arrived at after a study of different kinds of hammers and discussions with their manufacturers.

In the Report, the elastic compressions are stated as the terms $2\,C$, $2\,C_1$, and $2\,C_2$, and the sum of $C + C_1 + C_2$ is given as k. It would seem preferable to call the elastic compressions C_1, C_2, and C_3 and state Eq. 1 as:

$$0.5\,R_d\,(C_1 + C_2 + C_3) = R_d\,k \quad \text{.................... (32)}$$

if it is desired to use k at all, since the distance through which the head travels (which can be observed and measured in the field as hereinafter discussed) would then be the sum of C_1 plus C_2 and C_3. The factor 0.5 expresses the fact that the average travel of the cap, of the pile, and of the soil when compressed by the blow is only one half of the observed travel. It is an aid to clearer thinking concerning the meaning of the formula to keep these terms so that they will express the value to be checked by field observation. It would also seem better to state these terms as C_1, C_2, and C_3, instead of C, C_1, and C_2, as they are of equal relative importance to consider and the use of C for the cap appears to emphasize this factor unduly.

A-5. Hiley's Formula.—Eq. 4 states the applied energy as the product of the terms W and h. In the case of drop hammers and single-acting steam hammers, this is a simple procedure, as the catalogs state both figures. In the case of Items 4 and 6, Table 6, the rated energy (corresponding to W times h) is taken by the manufacturer as the sum of the energies obtained by multiplying

[20] "A Consideration of Pile Driving with Application of Pile Loading Formula," by Robert D. Chellis, *Journal*, Boston Soc. of Civ. Engrs., January, 1941, Vol. XXVIII, No. 1.

the weight of the ram by the stroke, plus the product of the piston area times the published steam pressure at the hammer. The maximum value is equal to the sum of the weights of the ram (W_r) plus casing (W_c), times the stroke. This is based on the theory that the steam pressure cannot exceed the weight of the casing without causing the casing to lift from the pile, or dance. In the

TABLE 6.—VALUES OF EFFICIENCY, e[a]

Item	Type of hammer	e
1	Drop hammers released by trigger	1.00
2	Drop hammers actuated by rope and friction winch	0.75[b]
3	Single-acting steam hammers	0.75[c]
4	Double-acting-compound California type steam hammers	0.65[c]
5	Double-acting steam hammers	0.85[c,d]
6	Differential-acting steam hammers	0.75[c,e]

[a] All of the percentages in this table have intentionally been given as slightly on the low side for all types of hammers (except the trigger-released drop hammers) compared to the values which might be used provided the hammers were all in excellent condition, in order to be reasonably certain of obtaining, at all times, approximately the intended energy as a minimum, since possible slight overdriving is preferable to underdriving. However, it is very likely that there are hammers in use today that do not deliver more than 50% to 60% of their rated energies.

[b] Item 2 may decrease when the drop is small or the drag considerable; and it may increase somewhat if the drop is very large or the drag not great.

[c] Items 3, 4, 5, and 6 include 0.10 to cover poor condition of hammer, wear, improper adjustment of valve gear, poor lubrication, unusual weather conditions causing condensation, unusually long hose, restricted areas at hose connections, minor hose leaks, unduly tight packing in types of hammers having manual adjustment of packing, un-noted minor drops in steam pressure which will reduce stroke, binding in guides, etc.

[d] Since the rated energies of a prominent make of the hammers under Item 5 are based on indicator diagrams, losses due to back pressure, pre-admission of steam before completion of the downstroke, expansion losses due to drop from entering pressure of steam, mean effective pressure in the cylinder, wire drawing of steam, and losses in valves and ports, have been deducted before obtaining the rated energies. The remainder are only mechanical losses such as those due to piston ring friction and tight packing to be covered by the value of "e." (The manufacturer recommends 90% as the lowest value of e for these hammers, even when operated under unfavorable circumstances.)

[e] Since the rated energies for Item 6 are based on the product of the entering steam pressure times the areas of the piston, the value of e should be such as to cover losses due to wire drawing of steam, piston ring friction, back pressure resulting from pre-admission of steam just prior to impact, losses in valves and ports, tight packing and other mechanical losses. It is claimed that the nonexpansive use of steam in the steam cycle in this type of hammer obviates a drop from the entering steam pressure to mean effective pressure. (A prominent manufacturer recommends 84% for the value of e for the large and medium-size hammers, and 80% for the small sizes, for hammers in first-class condition and operated under favorable circumstances.)

case of Item 5, Table 6, the rated energies (corresponding to W times h) stated in the catalogs of a prominent manufacturer are based on indicator diagram readings, confirmed by high-speed motion picture readings of the velocity of the ram at impact; and, although these values agree reasonably well with the theory of limiting the steam pressure by the weight of the casing, the variations are enough so that it is recommended that the rated energies be used, since the hammers act most efficiently at these figures.

In the second term of Eq. 4, representing impact loss, the W in both numerator and denominator represents the weight of the ram only, in the case of double-acting and differential-acting hammers. This may lead to confusion since in the case of Items 4 and 6, Table 6, the rated applied energy (corresponding to W times h) is obtained by the terms $(W_r + W_c)\, h$, whereas in the case of Item 5, Table 6, the rated energy should be taken from the catalog only. The energies for Items 4 and 6, Table 6, are also listed in the manufacturers' catalogs, which may be more convenient unless a table of casing weights is kept on hand. The writer finds a table of all ram and casing

weights, and strokes, of assistance.[20] It is recommended that Eq. 4 be subdivided into two formulas, as follows (retaining for the purpose of this discussion the terms C, C_1, and C_2 as proposed by the Report):

For use with drop hammers and single-acting steam hammers—

$$R_d = \frac{12\,e\,W_r\,h}{s + (C + C_1 + C_2)} \times \frac{W_r + n^2\,P}{W_r + P} \qquad (33a)$$

For use with double-acting and differential-acting steam hammers—

$$R_d = \frac{12\,e\,E_n}{s + (C + C_1 + C_2)} \times \frac{W_r + n^2\,P}{W_r + P} \qquad (33b)$$

in which E_n = manufacturer's rated energy (equals $(W_r + W_c)\,h$ in case of Items 4 and 6, Table 6); W_r = weight of ram; and W_c = weight of casing. It is recommended also that the term W be clarified to indicate exactly what weights are to be used, in which terms, in the formulas.

Eq. 4 is not best suited for the use of the engineer or inspector in the field, for computing carrying capacities of piles as driven. The writer uses a reduced form for field use, in which the office engineer has substituted the correct values for all terms except s and R_t. The factor of safety is selected, values of C, C_1, and C_2 are selected, and Eqs. 33 are solved for the desired penetrations. The formula for field use will then take the form:

$$R_t = \frac{X}{s + k} \qquad (34)$$

in which: R_t = safe design load in tons (due to dynamic resistance) to be assumed; X = numerical value for which Eq. 34 is to be solved; and s = penetration obtained by solving Eqs. 33, based on the allowable pile load multiplied by the factor of safety.

After X has been found and put in the formula as a constant coefficient, along with the other constant coefficient k, any value of R_t may be found readily in the field or office by using the various actual values of s that occur during driving. The caution should always be attached to each transmittal of such a formula that it applies only to the stated conditions attached, and that it will change with any change of equipment or pile types or lengths.

This reduced formula (Eq. 34) for field use, in the process of obtaining which, R_d is determined after assigning constant numerical values in Eqs. 33 to all terms except s, will apply only to penetrations within a reasonable range of the value of s determined when establishing the reduced formula, since with a considerably different s the change in value of R_d will change the values of C, C_1, and C_2 and thus affect the entire formula. For this reason it is often desirable to give the field office only the desired value of s to which the piles should be driven, and not the formula; however, if the penetrations obtained are within the same range as that specified, the reduced formula (Eq. 34) will furnish sufficiently close results. For penetrations widely different from that specified to correspond to the desired carrying capacity, the resulting R_d can be computed in the office.

For several years, the writer has kept a record of pile-driving cases with the corresponding derived reduced formulas (Eq. 34), the variations in the values of the numerator X and the temporary compression $2\,k$ being most interesting and instructive.[20] With the formula reduced back to this form, while including the results of considerations of the particular driving equipment and pile used, it is again in the simple form expressing the relationship, force $\times$ distance = force $\times$ distance. The sum of s plus $2\,k$ is the distance through which the top of the driving cap moves, and would also be the distance through which the tip of the pile moved if it were not for elastic losses in the cap, pile, and the quake of the ground. The term $2\,k$ is the distance in inches through which the top of the cap rises in rebound, leaving a net penetration at the tip due to the blow, of the distance s.

The writer has measured the amount of rebound in the field in several instances, near the head of the pile, and has found that the measured result checked the computed figures as closely as they could be measured with an ordinary rule. In these measurements C was not included, as it is difficult to take measurements at the top of the cap without special equipment; but since the error in C cannot be large compared with the total value of $C_1 + C_2$ (except possibly in the case of driving heavy mandrels or exceptionally heavy piles, with caps containing considerable packing), it is to the greatest advantage to check these terms. This can be done readily by holding a paper on the face of the pile just below the top, during the final driving, and moving a pencil continuously sidewise across the paper. The pencil should be guided on a horizontal stick. A graph is thus obtained of the total downward movement of the head of the pile under each blow, and the amount of rebound represented by the terms $2\,C_1$ and $2\,C_2$. Although it may be objected that the surface of the ground quakes, this is usually of small magnitude, and, for practical purposes, such measurements taken with an ordinary rule are adequate.

When computing C_1 it is important to consider whether the resistance is all at the tip of the pile, or whether the center of resistance is located at some distance from the butt. The value of L should be the distance to this point, and not necessarily to the tip. The field measurement of $2\,C_1 + 2\,C_2$, as described herein, serves as a check on the correctness of the assumed value of L, and if revisions in Eqs. 33 or 34 are found necessary after field measurements, they should be made.

A-8(a).—The Report takes a factor of safety of 3, instead of a purported factor of 6 as is done in the "Engineering News" formula. This is quite right, as a larger factor is not necessary to attempt to cover such a multitude of widely varying uncertainties and assumptions as are involved in that formula. The writer believes, however, that it is much better not to involve the factor of safety with the efficiency of the hammer in order to obtain different coefficients for drop and single-acting steam hammers, as shown in Eqs. 8 to 11. No formulas appear to be given to cover double-acting and differential-acting steam hammers.

The writer has studied the question of hammer efficiencies somewhat, and believes that the values stated herein under the comments on paragraph *A-4*

may be used as being reasonably within the range of truth. From those comments it will be noted that, with efficiencies varying from 100% to 50%, it would be better to retain this term, and not allow the variations to affect the factor of safety, which should remain a constant for any type of driving, piles, or hammer, in order to achieve the aim of securing comparable results. Actually, the sensitivity of the formula, due to the small proportion of applied energy that reaches the tip as driving energy, is very great, and a large difference in efficiency from that assumed may affect the results vitally.

When using the efficiencies suggested by the writer, it should be satisfactory to use a factor of safety of 2, 2.5, or 3 in usual cases, depending on conditions and the judgment of the engineer. This factor should be applied to the total ultimate carrying capacity, obtained after making such adjustments for friction as deemed proper.

Use of a factor of safety large enough to compensate for wide variations in efficiencies is erring again on the old path of the "Engineering-News" formula, in which practically all variables are intended to be covered by setting the factor of safety high enough to cover the worst cases.

By considering the efficiency of the hammer, the engineer will also gain a much wider appreciation of the capabilities and suitabilities of the various types of hammers, and will learn how widely the performance can vary, and possibly how it can be controlled to arrive at the best results.

Although the writer is averse to a complicated formula for field use, he believes that a very simple one can be set up for field use in every case, but that the engineer responsible for the work should set it up with full understanding of the various factors and their effects; otherwise he remains, as far as pile driving is concerned, essentially a "handbook" engineer.

The value of C_1 can be computed for any pile by the formula given in the Report. For ready reference, the writer keeps a small table of these values for all types of piles.[20] It should be noted that the material that is driven is that which should be considered—for instance, in the case of a concrete pipe pile, the pipe; or in the instance of a cased pile, the casing; and when driving a thin corrugated shell for a poured-in-place concrete pile, the mandrel. When computing the area of such a mandrel, the average area in square inches can be obtained by dividing the weight by the product of 3.4 times length of pile in feet. When a follower is used, an additional value should be obtained for C_1 to be added to the one obtained for the pile itself.

The value of E for steel sheet piling, steel tubular piles, pipe piles, fluted steel shells, and driving mandrels used inside thin corrugated shells, can be taken as 30,000,000. The value of E for pre-cast concrete is usually about 3,000,000, although it will vary with the consistency and age of the concrete. The value of E for wood piles varies with the kind of timber, whether green or air-seasoned, and is available from various sources.

The suggestion in the Report, that by taking L as the full length of the pile, compensation will probably be largely had for omitting the term C_2, throws the entire formula back into scarcely better case than the "Engineering News" formula, and defeats the opportunity to classify and study the mag-

nitude of the various energy losses and effective residue, and to take account of the enormously important variations in arrangement of soil strata. A logical study of these effects can scarcely fail to be both instructive and useful, as well as interesting.

The value of L should not always be taken as the full length of the pile, since the distance from the head of the pile down to the center of resistance determines the amount of elastic compression at the head of the pile, which is what the term $2\,C_1$ represents. Field measurements indicate that the total amount of temporary compression in the pile and the ground often is not as great as computed when using the full length of the pile for L, except for piles in pure end bearing. This is due to the fact that friction on the sides of the pile absorbs considerable energy before it can reach the tip. Probably if a reduced value of L were taken a closer agreement would be reached, giving consideration to the relative amounts of friction which it is judged the various strata can exert. This depends on the material, density, and depth of the various strata, and on the relative amounts of resistance which it is judged are obtained from friction and end bearing. For an end-bearing pile through soft material the value of L would be practically the full length. For a pile of which the lower half removed the load in friction plus some end bearing in cohesionless material, the value of L would be something more than three quarters of the length of the pile. If, in addition to this, there were to be an appreciable layer of sand above the soft material, a proportion of the driving resistance would occur in this stratum during driving, even though not considered as suitable permanent bearing capacity, and the value of L would have to be adjusted suitably. The actual center of resistance can be found readily by recording foot by foot penetrations during driving of the first pile, and computing the driving resistances at such locations as are necessary by means of the Hiley formula, and plotting the resistance curve. This is rarely necessary since the value of L usually can be selected with sufficient accuracy by inspecting the borings. Rankine's formula is based on using one half the pile length in all cases, but it is evident that this should not be done unless the entire length of the pile is in the load supporting stratum, which is not often the case. If a considerable amount of the pile or mandrel remains projecting above the ground when final penetration is reached, this fact should be considered when selecting L. With light-weight piles, which always have large values of C_1 in proportion to C and C_2, it is evident that an incorrect value of L would have particularly far-reaching effects on the result. The value of $2\,C_1$ may be visualized by imagining driving on a spring, and the more flexible the spring, the greater temporary compression.

A-8(b).—The Report recommends that the value of C be taken as 0.05 for medium driving resistance (1,000 lb per sq in. at the point of the pile). When computing the stress at the head, the writer uses the area of the head of the pile when selecting the value of C, since the compression of the cap varies with the area, the force being constant, and thus a great difference occurs between the caps on the heads of straight-sided and tapered piles. The writer uses the following values of C for a stress of 1,000 lb per sq in. on the head of the pile, based on the ultimate driving resistance R_d:

Description	Values of C, in inches
Head of timber pile	0.05
3-in. to 4-in. packing inside cap on head of pre-cast concrete pile	0.05 to 0.07
$\frac{1}{2}$-in. to 1-in. mat pad only, on head of pre-cast concrete pile	0.025
Steel covered cap, containing wood packing, for steel piling or pipe	0.04
$\frac{3}{16}$-in. red electrical fiber disk between two $\frac{3}{8}$-in. steel plates, for use with severe driving on fluted steel pile	0.02
Head of steel piling or pipe	0

For easy driving (500 lb per sq in.) on the head, reduce the foregoing values 50%; for difficult driving (1,500 lb per sq in.), increase them by 50%; and for extremely difficult driving (2,000 lb per sq in.), double them.

In piles having a large area, such as pre-cast concrete, or a steel mandrel, the value of C_1 is very small, and in such a case a flat value of 0.05 in. for C may overshadow the amount of C_1. This would affect the value of R_d materially, and distinction should be made between the various types of caps and piles, in order to arrive at better comparable results, which should be the object sought.

In the case of anvils having a cup to receive a driving block, common practice is to throw in short pieces of wood, from time to time, which may compact to a hard layer, with the result that the value of C may vary widely, and although this will not matter until the pile approaches final penetration, at that time an effort should be made to obtain consistency in the results to approximate the value of C used in the formula. This is particularly important when the value of C may be large relative to the value of C_1 plus C_2.

A-8(c).—When selecting the value of C_2 for temporary compression of the soil, account should be taken of the compressibility of the strata resisting the blow, including that below the pile tip. In the case of a soft bed under the stratum in which the tip rests, a larger value would be used than if a hard material were present.

The Report states that the value of C_2 is ordinarily indeterminate and is neglected; and that it is probably largely compensated for by using the full length of pile for L. The effects of considering L in this manner have been discussed. In the case of heavy piles such as pre-cast concrete or steel mandrels, where the value of C is very small, neglect of the term C_2 may cause a great difference in the answer. To understand the relative effect of each loss it is much better to compute them separately, as will be described in this discussion, and retain each term. As described, it is easy to measure the sum of $2\,C_1$ plus $2\,C_2$ on the first pile driven, and readjust the specified results accordingly by a simple computation if this is found necessary. By subtracting the computed value of $2\,C_1$, from the measured amount, the magnitude of $2\,C_2$ can be determined for that particular soil and case. A record of some such cases will soon provide the engineer with a guide to the values which may be expected. Until such figures are accumulated, the values given by Hiley may be used. (The writer has tabulated values of C_1, C_2, and C_3, which in the Report would be called $2\,C$, $2\,C_1$, and $2\,C_2$, respectively, due to a slight difference in terminology.)

A-8(d).—A far better understanding of the relative actions of the head, pile, and soil under different conditions can be obtained through the thought required to select each coefficient individually, than by the use of a table of k-values. The k-values ignore many relatively important items such as type of cap, effective length of pile to its center of resistance, character of soil, etc., and no intelligent handling of the problem is possible if such matters are not considered.

A-13 and A-14.—Approximate Static Formulas.—Although no great amounts of published data are available regarding actual friction values, and it does not seem best to design piles entirely on this basis, nevertheless this aspect of the matter should be considered and used as a check or aid to judgment of the pile-driving requirements.

A certain amount of skin friction is present on the pile during driving and the energy of the hammer blow is not all transmitted to the tip of the pile. In cases where the entire lengths of the piles are in cohesionless materials, such as sands and gravels, or where the bottom or carrying stratum consists of such materials, it is immaterial whether the desired resistance is obtained by end-bearing or by skin friction, and the value of L in the formula should be judged in accordance with conditions. There will not be much, if any, increase in friction over a period of time for the parts of piles driven in these materials. If any considerable part of the pile is driven through cohesive materials, such as clays, after the value of R_d is obtained, an allowance for the effect of friction may be made. These effects may be either additive or subtractive, in accordance with soil conditions.

Where upper strata are present that are capable of picking up friction load from the pile and distributing it on material able to carry it without greater settlements than strata below, the carrying capacity of the pile increases after driving. Where a pile is driven through a hard upper layer, then through a soft layer and into a sand and gravel stratum, the friction on the upper layer might cause the upper layer to put an additional load on the pile as the soft layer settled. For many cases, consideration will show that friction computations are unnecessary. This is true in the case of piles driven through deep upper strata consisting of very soft material, which can neither exert much friction during driving, nor be set up to cause much load by friction after driving. They can only be driven a relatively short distance into a firm stratum, so that they act essentially in end bearing. It is often sufficient, particularly on small jobs where the difference that might be involved in total pile footage is inconsiderable, merely to bear in mind the possible effects of friction when selecting the factor of safety.

No method of expressing friction setup occurs in Eq. 4, but the article by Mr. Hiley,[1] discusses this matter at some length, and gives examples and methods of computation. Mr. Hiley recommends considering that all of the friction occurring in cohesionless soils, and one half of that occurring in cohesive soils, be considered as operative during driving and consequently included in the driving resistance. Hence, to arrive at the total bearing value of the pile,

[1] "Pile-Driving Calculations with Notes on Driving Forces and Ground Resistance," by A. Hiley (Theory and practice; table of forces transmitted through pile; energy requirements; bearing qualities of ground), *Structural Engineer*, Vol. 8, 1930, pp. 246–259, 278–288.

account must be taken of these quantities of friction and of the remaining friction that will occur during the setup of the cohesive strata around the pile after driving. This setup commences very soon after driving, and will be noticed if piles are redriven, by observing that the penetrations are smaller than those that occurred at the cessation of driving.

After the pile lengths have been selected tentatively, but before ordering piles, it is advisable to consider the total lengths of embedment in load-resisting friction strata and the resulting carrying capacities strictly from a friction point of view, after, however, deducting such percentage for point resistance as it is felt certain will be present, as a rough check upon the designed lengths.

A table of representative recorded friction values selected from various sources, more complete than that contained in the Report, has been published elsewhere.[20] There is a great need of further data of this nature.

B-1.—The cautions contained in this paragraph are of great importance, and are too often overlooked. Soil mechanics now furnishes a tool by which these matters may be considered.

B-3. Dynamic Formulas.—It is true that there are a great number of dynamic formulas; but practically all that are in use are the same formula, with various terms omitted or approximated. The derivation of the entire series of Hiley, Terzaghi, Redtenbacher, Rankine, Dutch and "Engineering News" formulas, representing the same basic equation of "force times distance" equal "force times distance, less efficiency, impact and energy losses," was most interestingly presented by J. Stuart Crandall in 1931.[21] A study of these derivations is most useful.

B-6. Reliability.—The statement that a complicated formula is not recommended, since such formulas have no greater claim to accuracy than the more simple ones, does not appear logical, and is not borne out by the writer's experience. By making the application of the Hiley formula simple, in the manner outlined by the writer, very uniform results have been obtained. Examples of this fact, and the distribution of the energy losses neglected or incorrectly approximated in the older formulas, are afforded by the comparative test results described in this discussion. The Committee also states that a dynamic formula is nothing more than a yardstick to help the engineer secure reasonably safe and accurate results over the entire job. Is any formula capable of being more than this? Piles must always be driven with human judgment, and a satisfactory yardstick is the thing to be desired. By an understanding of the elements entering into driving, as outlined in this discussion, the writer believes that the Hiley formula furnishes an excellent yardstick.

It is possible that the theory of longitudinal impact will also furnish a good yardstick, but literature and data are not sufficiently available as yet to enable judgment to be formed. This formula appears most complicated, with many wide assumptions required in order to apply to pile-driving conditions.

B-10.—It is desirable to check the temporary elastic compression $(2\,C_1 + 2\,C_2)$ by field measurements, but office assumptions should be satisfactory for preliminary requirements, at least. It has been the writer's experience to date

[21] "Piles and Pile Foundations," by J. S. Crandall, *Journal*, Boston Soc. of Civ. Engrs., May, 1931, Vol. XVIII, No. 5.

that measured values of the temporary elastic compression have agreed closely with the computed values. This may be good fortune, and does not change his opinion that it is desirable to have a field check on these figures.

The value of C is difficult to measure, and need not be checked unless it is large compared to C_1 plus C_2. If measured, or if important, more control over its variations should be kept than is the customary practice; or, a driving criterion should be set up which will bracket the limits of its variations, in which case no measurements or control of C will be necessary.

The recommendation that the energy of the hammer be checked in the field is a difficult one to follow in the case of double-acting and differential-acting steam hammers. Since the energy varies with the speed, and since the energies corresponding to various speeds are stated in the manufacturers' catalogs, it is more practicable to select the energy from this source. In this case, the efficiencies given in the writer's discussion of Paragraph *A-4* are recommended, instead of using the manufacturers' rated energies directly, however. The rated energies for these different types of hammers are not comparable, since for double-acting hammers the energies are based on indicator diagram readings and have been closely checked by moving picture records of the velocity of the ram. Therefore they are the net effective energies remaining after the deduction of losses to this point, whereas, for differential-acting hammers the rated energies are based on the product of the entering pressure times the area of the piston, and therefore losses beyond the entering port must be covered by the efficiency coefficient. The writer believes that the use of these rated energies in conjunction with the efficiencies listed herein will be much more practical and satisfactory than endeavoring to measure the energies.

The following pages contain an example of the writer's method of applying Eq. 4, a study of the comparative results obtained by the use of this formula and other well-known formulas, applied to actual test cases, a description of the method of using this formula for investigating existing piles of which the driving records are available, and other comments that did not directly fit into the numbered paragraphs in the Report. Many of the points mentioned are those upon which pile-driving operations "go wrong," for lack of instructions or information upon the subject, and should be embodied in the manual.

Analysis of Energy Losses and Check of Computations.—After obtaining the value of R_d, it is advisable to ascertain the parts of the total applied energy lost due to various causes when driving the pile, in order to observe the efficiency of the hammer, and as a check upon the computations, by substituting the value of R_d in one of the following formulas:

For use with drop hammers and single-acting steam hammers—

$$R_d = \frac{12\,e\,W\,h}{s} - \frac{12\,e\,W\,h\,P\,\dfrac{(1 - n^2)}{W + P}}{s} - \frac{R_d\,C}{s} - \frac{12\,R^2_d\,L}{A\,E\,s} - \frac{R_d\,C_2}{s} \tag{35a}$$

For use with double-acting and differential-acting steam hammers—

$$R_d = \frac{12\,e\,E_n}{s} - \frac{12\,e\,E_n\,P\,\dfrac{(1 - n^2)}{W + P}}{s} - \frac{R_d\,C}{s} - \frac{12\,R^2_d\,L}{A\,E\,s} - \frac{R_d\,C_2}{s} \tag{35b}$$

in which R_d = the ultimate carrying capacity = the ultimate resistance to driving. For convenience of discussion consider Eqs. 35 in the generalized form:

$$R_d = \text{Quantity } A - \text{Quantity } B - \text{Quantity } C - \text{Quantity } D - \text{Quantity } E \qquad (36)$$

Then the significance of the various quantities is as follows:

Quantity	Definition
A =	The ultimate carrying capacity without losses = the kinetic energy applied, divided by the penetration under the last blow;
B =	The loss in ultimate carrying capacity due to impact = the impact loss divided by the penetration under the last blow;
	The loss in ultimate carrying capacity, due to the energy loss in imperfectly elastic compression, of:
C =	the pile head and cap = $\frac{R_d C}{s}$ = the energy loss in the pile head and the cap, divided by the penetration under the last blow;
D =	the pile = $\frac{12 R^2_d L}{A E s}$ = the loss in the pile divided by the penetration under the last blow; and
E =	The soil = $\frac{R_d C_2}{s}$ = energy loss in the soil divided by the penetration under the last blow.

Eqs. 35 are of assistance in selecting the proper weight of hammer to use, as the proportion of useful energy to wasted energy may be observed. For economical and efficient driving a reasonable proportion of the applied energy should remain to produce the force R_d. Furthermore, if the remaining force is very small, slight uncertainties in the assumptions may be as great numerically as the value of R_d, indicating that the sensitivity of the formula in this region is too great to place too much reliance on the computed results in such cases. Both for reasons of economy and in the interest of reliance on the computed results, a hammer of efficient size should be used. It is better to select a hammer that may be on the heavy side, rather than one on the light side. In general, the hammer should be as large as can be used safely without damaging the pile, and the computed stress in the pile head should be compared with the yield point of the pile material to be sure that this value is not exceeded. In several instances of yielding of the pile head, it has been observed that the stress computed in this manner has been practically equal to the yield point of the material.

Sample Computation for Penetrations.—Consider a double-acting steam hammer (Item 5, Table 6) with a capacity of 140 blows per min. The energy, E_n, is 8,200 lb-ft; the cap weighs 650 lb; and the pile is a steel pipe $\frac{1}{4}$ in. thick, with an outside diameter of 10 in. and a length of 30 ft (to be filled with concrete after driving). The pile weighs 790 lb. The soil conditions are as follows: 5 ft of top soil and recent fill, 20 ft of soft clay, 2 ft of sand, 20 ft of gravel, and 5 ft of hard pan over rock.

Let: $P = 790 + 650 = 1{,}440$ lb; $W = 1{,}500$ lb (ram only); $e = 0.85$; $n = 0.4$: $R_t = 30$ tons (working load); factor of safety (assumed) $= 2.5$; $R_d = 30$ tons $\times$ 2,000 lb $\times$ 2.5 = 150,000 lb; $C = 0.08$ in. (the stress on the cap being $\dfrac{150{,}000 \text{ lb}}{78 \text{ in.}^2} = 2{,}000$ lb per sq. in.); the net area of the pile = 9.87 in. $\times$ 3.1416 $\times$ 0.25 in. = 7.73 sq in.; and

$$C_1 = \frac{150{,}000 \times 30 \times 12}{2 \times 7.73 \times 30{,}000{,}000} = 0.12.$$

By Eq. 33b: $150{,}000 = \dfrac{0.85 \times 8{,}200 \times 12}{s + (0.08 + 0.115 + 0.025)} \times \dfrac{1{,}500 + 0.4^2 \times 1{,}440}{1{,}500 + 1{,}440}$

$= \dfrac{83{,}500}{s + 0.22} \times 0.589$; from which, $s = 0.328 - 0.22 = 0.108$ in. (9 blows per in.).

Sample Computation for Analyzing Force Losses.—By Eq. 35b, and using the data in the foregoing example:

Computation (see Eq. 36)		Losses	Capacity
$A = \dfrac{0.85 \times 8{,}200 \times 12}{0.108}$	=		775,000
$B = \dfrac{1}{0.108}\left[0.85 \times 8{,}200 \times 12 \times \dfrac{1{,}440\,(1 - 0.4^2)}{1{,}500 + 1{,}440}\right]$	=	− 318,000	
$C = \dfrac{150{,}000 \times 0.08}{0.108}$	=	− 111,000	
$D = \dfrac{150{,}000^2 \times 30 \times 12}{7.73 \times 30{,}000{,}000 \times 0.108}$	=	− 161,000	
$E = \dfrac{150{,}000 \times 0.025}{0.108}$	=	− 35,000	− 625,000
R_d = a check on the assumed value	=		150,000

Short Formula for Field Use.—The short formula for field use may be obtained as follows by the application of Eq. 34: 30 tons $= \dfrac{X}{0.11 + 0.22}$; and $X = 9.9$. Therefore, $R_t = \dfrac{9.9}{s + 0.22}\left(\text{use } R_t = \dfrac{10}{s + 0.22}\right)$.

Investigation of Load Capacity from Known Penetration.—In the case of an investigation in which the penetration is known and it is desired to find the corresponding value of R_d, it is first necessary to assume a value of R_d, for the purpose of obtaining C_1 for use in Eqs. 1 and 4 by means of Eq. 10, and if the value of R_d then obtained varies from the assumed value, a new trial should be made. It is also necessary to assume values of C and C_2 corresponding to the value of R_d. Three or four trials usually suffice to determine R_d.

Comparative Results from Actual Tests.—In order to illustrate the value of Eqs. 33 in obtaining comparable results when reaching a given depth or strata, and the effect on the question of cost when determining pile lengths, the following cases obtained from actual driving operations are presented:

Case A.—In this case three different types of piles were driven with the same hammer near a lake shore. Eleven piles were driven near each other in the same strata with a single-acting steam hammer having a 5,000-lb ram and an observed 34-in. stroke. Three piles were standard thin corrugated-shelled piles 29 ft 4 in. long, driven with a heavy steel mandrel; three were No. 3 gage fluted tapered steel shells 25 ft long with 8-in. tips and $14\frac{1}{4}$-in. butts, filled with high-early strength cement six days before driving; three were No. 3 gage fluted tapered steel shells 25 ft long with 8-in. tips and $14\frac{1}{4}$-in. butts; and three were No. 7 gage fluted tapered steel shells 25 ft long with 9-in. tips and $15\frac{5}{8}$-in. butts. Soil conditions consisted of 4 ft of cinder fill on 2 ft to 4 ft of marsh over fully inundated noncohesive lake sand. The ground-water level was approximately 5 ft below the surface. A pit about 2 ft deep was dug for each pile to remove a frozen layer of the fill.

A driving cap about 11.5 in. in diameter, consisting of a 2-in. steel plate over a 6-in. hardwood block over a 1-in. steel plate, weighing about 150 lb, was used on each pile. In addition, a 4-in. hardwood block was placed on the concrete of the prefilled piles under the lower plate. The tapered fluted steel piles were all driven with a follower in addition, consisting of a piece of steel mandrel weighing 540 lb.

The weights of piles, mandrels, caps, and followers were as given in Table 7.

TABLE 7.—WEIGHTS (IN POUNDS) OF PILES, MANDRELS, CAPS, AND FOLLOWERS

Pile	Mandrel	Pile	Cap	Follower	Total weight (P)
Piles with thin corrugated shells	9,425	250	150	....	9,825
Fluted steel prefilled shells, 8-in. tips	....	2,560	165	540	3,265
Fluted steel shells, 8-in. tips	....	780	150	540	1,470
Fluted steel shells, 9-in. tips	....	600	150	540	1,290

TABLE 8.—SAFE BEARING VALUES (IN TONS) BY VARIOUS FORMULAS

Type of pile	Penetration (in.)[a]	Equations 33*a* and 35*a*; Quantities (See Eq. 36):						New working load[b]	Eq. 5	Eq. 37*a*	Eq. 37*b*	Eq. 37*c*
		$+(A)$	$-(B)$	$-(C)$	$-(D)$	$-(E)$	R_d					
Piles with thin corrugated shells	0.125	544	323	85	8	21	107	43	63	45	33	78
Fluted steel prefilled shells, 8-in. tips[c]	0.26	260	91	46	14	11	98	39	39	45	25	46
Fluted steel shells, 8-in. tips	0.30	227	47	32	34	8	106	42	35	43	24	44
Fluted steel shells, 9-in. shells	0.23	296	55	46	71	12	112	45	43	58	27	58

[a] Average values. [b] Using a factor of safety of 2.5. [c] Assuming $n = 10$ for concrete.

The safe bearing values in Table 8 were computed from the observed penetrations at tip elevations 15 ft below ground surface. Comparisons are offered between the results by Eqs. 33*a* and 35*a*, the "Engineering News" formula

(Eq. 5), Eytelwein's formula:

$$R_d = \frac{2\,W\,h}{s + 0.1\,\frac{W_p}{W_r}} \quad \text{(37a)}$$

the formula frequently used for single-acting steam hammers by the Bureau of Yards and Docks, U. S. Navy Department:[22]

$$R_d = \frac{2\,W\,h}{s + 0.3} \quad \text{(37b)}$$

and the so-called Navy-McKay formula:

$$R_d = \frac{2\,W\,h}{s\,(1 + 0.3)\,\frac{W_p}{W_r}} \quad \text{(37c)}$$

The term W_p in Eqs. 37*a* and 37*c* was taken as the weight of the pile only, in accordance with customary practice. The point was brought out in this discussion that in the Hiley formulas the term P, or W_p, should include cap and follower weights with the pile weight.

In computing the results by means of Eq. 33*a*, values of L were taken as the distances from the heads of the piles to a point at half the distance of embedment in the sand stratum; the value of e was taken as 0.8; and the value of C_2 as 0.025.

From Table 8, it appears that Eq. 33*a* results in a maximum difference of only about 13% in bearing values for the different types of piles. If some allowance were made for end bearing during driving, and the value of L was taken somewhat longer, the load-carrying capacities of the more compressible piles, such as the empty fluted steel shells, would decrease by a ton or two, and even closer agreement would be noted between the various kinds of piles. These agreements are close enough for all practical purposes.

In order to determine the depths to which the fairly light-weight fluted steel shells would have to be driven (needlessly) to give the same indicated bearing value by the "Engineering News" formula as the 63 tons shown for the piles driven with the heavy steel mandrel, it was necessary to observe the tip elevations on the driving records at which penetrations of 0.125 in. per blow were obtained for these piles. It was necessary to drive the tips of the 8-in. fluted steel shells 22.5 ft below ground level in order to obtain this penetration, thus having the effect of increasing the length of all the piles on the job by 50%. This added length is of no practical value, since the 15-ft length was sufficient to transmit the load safely into the bearing stratum of sand. It will be noted, however, that by use of Eq. 33*a*, practically identical depths of penetration would be required for any of the types of piles to obtain the same safe bearing value.

Case B.—A further comparison is given by the following case in which three light fluted steel shells, 40 ft long, having 8-in. tips and a taper of 1 in. in 4 ft, and five thin corrugated steel shells 31.5 ft long, having 8-in. tips and a taper

[22] "Pile Formula Modified for Double-Acting Hammer," *Engineering News*, Vol. 76, No. 1, 1916, p. 29.

of 1 in. in 2.5 ft, were driven in the same strata by a single-acting steam hammer having a 5,000-lb ram with a stroke of 36 in. A fluted steel shell pile weighed 600 lb; the "bath-tub" follower, 2 ft long, weighed 800 lb; and the driving cap, consisting of a hardwood block 11.5 in. in diameter by 6 in. thick, with a 2-in. steel plate on top and a 1-in. steel plate below, fitted into a shield, weighed 225 lb. A steel mandrel for driving the thin corrugated shells weighed 12,300 lb and the 225-lb cap only was used with it. The strata consisted of 10 ft of cinder, gravel and ash fill, 5 ft of fine sand, 11 ft of peat and sand, 8 ft of loose silty sand, 4 ft of coarse sand and gravel in which the final tip readings were taken, and below that 6 ft of medium gray clay, 12 ft of firm fine sand and a little clay, and a deep bed of firm fine sand. The comparative results are given in Table 9(a), showing the safe bearing values computed by Eqs. 5 and 33a.

TABLE 9.—COMPARATIVE RESULTS (IN TONS) OF COMPUTATIONS BY EQS. 5 AND 33a

Type of pile (Table 9(a)) or hammer (Table 9(b))	Penetration, in inches[a]	EQS. 33a AND 35a; QUANTITIES (SEE EQ. 36):							Eq. 5
		A	$-(B)$	$-(C)$	$-(D)$	$-(E)$	R_d	Net working load[b]	
(a) DIFFERENT TYPE OF PILE DRIVEN WITH THE SAME HAMMER									
Fluted steel shells	0.85	79	17	2	10	3	47	19	16
Thin corrugated shell piles driven with steel mandrel	0.375	180	115	2	1	11	51	20	31
(b) SAME TYPE OF PILE WITH DIFFERENT HAMMERS									
Double-acting steam hammer, rated energy 5,400 ft-lb[2,c]	0.116	238	92	0	52	17	77	31	29
Differential-acting steam hammer, 5,000-lb ram	0.563	120	22	0	14	4	80	32	23

[a] Average value. [b] For a factor of safety of 2.5. [c] At 120 blows per min.

The value of L in computing C_1 for the fluted steel piles was taken as 30 ft, being based on a center of resistance about two thirds of the distance from the ground surface to the tip, and for the 37-ft 4-in. long steel mandrels as 20 ft, being based on a center of resistance about half the embedment of the casing, on account of the greater taper. Due to the small elastic compression loss in the heavy steel mandrels it makes practically no difference as to the value assumed for L for them. The value of C_2 was taken as 0.05 for driving the fluted pile shells, and 0.075 for the steel mandrels.

In order to secure for the fluted pile shells an equal 31-ton capacity by the "Engineering News" formula it was necessary to drive them an additional 15 ft. On the other hand, the pile tips driven with the heavy mandrel stopped just short of the 6-ft bed of clay, and the fluted shell pile tips carried through this clay into the firm sand below. The stratum in which the tips should rest should be determined from the borings, however, and not be governed by finding adequate driving resistance just above the clay, if detrimental settlement

of the clay bed is expected. In Case A it appeared that use of the "Engineering News" formula required the fluted pile shells to be driven to a greater depth than actually necessary, whereas in Case B it would appear that use of the "Engineering News" formula, without consideration of the relation of the tip elevation to the strata, would result in stopping the mandrel-driven piles at too high an elevation.

Case C.—Cases A and B have illustrated the computation of safe loads in the cases of different types of piles driven with the same hammer. Case C shows the computation of safe loads for a case in which the same type of pile was driven with two different kinds of hammer. The piles were 8-in., 36-lb H-beams, 46 ft long, driven to a length of 44.5 ft in the ground. The strata consisted of 4 ft of soft clay with some sand, 19 ft of soft mud with some sand, 8 ft of soft mud and fine sand, slightly firmer than above, 5 ft of fine silty sand and mud, 10 ft of silty coarse sand and gravel, 7 ft of coarse sand and gravel in which the tips rested, and coarse sand and clay below. Seven piles were driven with a double-acting steam hammer and nine piles with a differential-acting steam hammer having a 5,000-lb ram. The speed of the double-acting hammer was observed as only 120 blows per min, at which speed the rated energy is 5,400 ft-lb. The value of L for use in determining C_1 was taken as 27 ft. The value of C was taken as zero, since no cap was used and the piles were steel. The value of C_2 was taken as 0.025 in both cases. The comparative driving results are given in Table 9(b).

Case D.—Case D illustrates the correspondence of the computed and measured value of C_1, and of the theoretical and actual yield points of the piles. Piles consisting of 10-in., 42-lb H-beams, 50 ft long, were driven through soft material consisting of 10 ft of slag and cinder fill, 19 ft of river muck, 20 ft of sand, clay and gravel, and 1 ft or 2 ft of weathered shale or hard clay to hard shale rock, with a single-acting steam hammer having a 7,500-lb ram and stroke of 42 in. The weight of the driving cap, containing a wood block, was 700 lb. The ultimate driving resistance equaled the theoretical yield point of the pile, and was 406,000 lb. The computed elastic compression C_1 at this load was 0.67 in. This condition should have occurred when driving resistance reached approximately 30 blows per in. Due to the solidity of the rock resistance compared with any frictional resistance, the value of L was taken as the full length of the pile. These results were checked by field observation, a value of $\frac{5}{8}$ in. being measured for C_1, and failure of the pile metal occurring when driving required slightly more than 30 blows per in. As ultimate bearing values are reached, the formula indicates that the number of blows per inch increases rapidly, with almost no increase in the value of R_d.

Speed of Hammer.—It is particularly important that double-acting and differential-acting steam hammers be run at the full listed speeds, as the net available energy at the pile tip falls off rapidly at lesser speeds, particularly sharply in the case of undersized hammers. The number of strokes per minute occurring when the observer is taking final penetration readings should always be noted on pile-driving reports, particularly if, for any unavoidable reason, the speed is less than the maximum specified, for if the speed has decreased and is

not noted, the energy is reduced, and the smaller penetrations obtained will indicate falsely high bearing values. The writer has often endeavored to draw conclusions from old pile-driving reports, which would be of assistance in determining the driving criteria for new work to be performed in accordance with the methods outlined herein, and has been unable to place any faith in the old reports, for lack of notations regarding the hammer speeds, and with the fact in mind that it is, and probably has been more so in the past, fairly common practice to pay very little attention to the hammer speed or its variations. Many pile inspectors are not fully aware of the theory behind pile driving, beyond the use of the "Engineering News" formula, and therefore the specifications should emphasize this point, and the blanks provided for pile-driving reports should contain a space to be filled in regarding the speed of the hammer during final penetration.

Stroke.—In the case of single-acting steam hammers the actual stroke should be measured, at the time close to final penetration. Strokes are frequently less than the nominal value, and due to the sensitivity of the formula, when the net force at the tip is small compared with the applied force, a reduction of a few inches in the stroke with consequent reduction of applied energy may make a large percentage reduction in the net energy after deducting losses during driving. The inspector should note the stroke on the reports, and if less than the normal, should immediately call the matter to the attention of the engineer.

Redriving.—Cases have occurred in which pile-carrying capacities have been interpreted on the basis of results from redriving after considerable intervals, or after shutdowns for lunch or repairs, instead of at the close of original continuous driving to the required depth. Specifications should make clear that carrying capacities are not to be computed in this manner. The earth is likely to set considerably around the pile, more particularly with the clayey or silty type of strata, which will affect the results markedly, and no allowance for this condition appears in Eq. 4. A body of data can be built up, and judgment developed, on the basis of continuous driving, but a "setup," of varying times and amounts, would render the interpretation of such experience practically impossible. However, there are times when the engineer can learn much by requiring redriving at stated intervals, in order to aid his judgment in determining the amount of added friction, either useful or detrimental according to the location of the strata, on the pile.

If for any reason it has been necessary to suspend driving of a pile before final desired tip grade has been reached, the tip grade at this time and duration of the delay should be noted on the pile-driving reports, together with the reason, and the pile driven to a tip grade comparable with adjacent piles.

Batter Piles.—When driving batter piles the height h in the formula is reduced, and also friction occurs in the guides in the case of single-acting and drop hammers. Taking the coefficient of friction as 0.1, the effective value of drop h_1 to be used in Eq. 4 may be taken as follows (in which θ = the angle between the batter and the vertical):

For drop hammers and single-acting steam hammers—

$$h_1 = h\,(\cos\theta - 0.1\sin\theta) \qquad \text{(38a)}$$

For double-acting and differential-acting steam hammers—

$$h_1 = h \cos \theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38b)$$

Underwater Driving.—When driving under water with suitable types of hammers, such as double-acting steam hammers or closed type differential-acting steam hammers, suitable compensation must be made for the buoyant effect of the water by hanging sufficient equivalent dead weight on the hammer casing; otherwise the energy of the blow will be reduced.

Uplift Piles.—Piles are sometimes required to resist hydrostatic, or other static, uplift on the structure, by means of friction, when the weight of the structure is insufficient to prevent flotation or uplift. For such purposes the lengths of piles should be predetermined by means of selected safe friction values[20] for the various strata, or pulling tests. The factor of safety selected need not be large, and depends somewhat upon the importance of the structure, frequency of the uplift, and expected duration of the uplift.

The total weight of earth clinging to the piles in uplift should also be considered, since regardless of the magnitude of the friction value on a single pile, the piles cannot pick up more earth than is adjacent to the group and above their tips.

The design of uplift piles should provide for tension in the pile material, and suitable provision should be made for transmitting the tension into the structure.

The foregoing discussion of uplift piles is based on straight-sided or slightly tapered piles. It would not normally be advisable to select piles with a great deal of taper for uplift purposes. Generally the type of pile with the largest perimeter should be chosen for uplift. Sometimes core-stoppers are used in the web spaces of H-piles, but there is contention as to whether or not their use adds to the pulling value of the pile. It is generally assumed that the effective friction surface of an H-pile is equal only to the outside bounding dimensions, and that the soils in the core become firmly wedged in place, in which case no added value would be computable from core-stoppers. Tests have also indicated that piles displacing all of the earth within their perimeter have a much greater friction value than piles that do not do so. This point should be borne in mind when considering friction values.

There are other types of piles than pure friction piles, however, which may be considered for uplift resistance, such as button-bottom piles, and bulb-bottom piles in which a bulb of concrete is forced out of the bottom of the casing by ramming. In the case of button-bottom piles the driving of the button forms a hole of that size in the ground, and consideration must be given to the extent and rapidity to which it is expected that the ground will close in against the pile. In the case of bulb-bottom piles a large volume of soil must be lifted in order for the pile to pull out. Possibly the volume of soil to be lifted might be considered that included within a truncated cone having sides stopping at an angle of 30° with the vertical.

Correction: Change the last line of Paragraph *A-8(d)* to read: "* * * capacities are available."

Lazarus White,[23] M. Am. Soc. C. E.[23a]—The writer has been a member of the Committee on the Bearing Value of Pile Foundations and can testify to the conscientious work of this Committee over a period of many years.

Despite the painstaking work of the Committee, under the able direction of its Chairman, little progress has been made toward the main purpose—the bearing value of pile foundations. As the "Foreword" states,

> "It must be emphasized urgently that the total of the bearing value of individual piles, no matter whether determined by formula or by load test, is not the bearing value of a pile foundation as a whole. The problem in regard to the pile formula is to determine the bearing value of an individual pile. It must be stated further that the bearing value of a pile determined from the penetration per blow of the last few blows, or even the bearing value determined from the load test, is the value only at the time that the data were obtained. The bearing value of the pile under certain conditions may be something quite different 24 hr later."

That being the case, why so much emphasis (Report *A*) on a new pile-driving formula which at best can only approximate the value of a single pile and gives no value for a pile foundation composed of a group of piles?

The proposed formula (Report *A*) has the failings of all previous pile formulas—it can give only the value at the time of driving (if it can do even that) and not 24 hr later. The proposed formula has the basic form of the "Engineering News" formula. The writer repeatedly has compared the results of the "Engineering News" formula with actual tests on piles hydraulically loaded, and has found the widest variations between actual test loads and the loads derived from the "Engineering News" formula. G. G. Greulich, Assoc. M. Am. Soc. C. E., a member of the Committee, has compiled a large number of tests on piles and compared them with the theoretical loads obtained from the pile-driving formula, and has found the widest variations. Moreover, the writer's firm has repeatedly underpinned structures that should not have suffered from settlements—were the "Engineering News" formula reliable.

The writer can see no advantage in advocating the use of the complicated formula in Report *A*. Consider the coefficients which must be determined to use this formula: Weight of pile as driven; modulus of elasticity of pile as driven; cross-sectional area; "coefficient of restitution"; "efficiency of hammer"; "elastic compression of driving cap"; and the rebound of the pile hammer.

Of course, by choosing a value for the "coefficient of restitution," which, in the tabulation in Paragraph *A*-7, varies between 0.0 and 0.5, together with skilfully choosing the values for the numerous other coefficients, the engineer might compute a value approximating that given by a loading test.

It seems that the proposed formula (Report *A*) is derived from Hiley's formula, presented in 1930. As far as the writer knows the formula has been used little—certainly it has not been checked sufficiently against actual tests and experience to deserve the official approval of the Committee.

Furthermore, it would be a calamity for the Society to lend its authority to the promulgation of any pile-driving formula as yet described. With this

[23] Pres., Spencer, White & Prentis, Inc., New York, N. Y.

[23a] Received by the Secretary August 23, 1941.

backing the formula would be as blindly used as previous formulas, a serious setback to the science of foundations.

The writer does not wish to disparage the work of the Committee; he merely believes that there was an excess of zeal in preparing a pile-driving formula, and that, in view of the disagreement in the Committee itself, no formula should have been proposed; and that the disagreement should have been clearly reported. It would have been a great service to the engineering profession if the Committee had reported that further progress in pile foundations cannot be made by "polishing up" pile-driving formulas. However, the engineer does not need to stay in the "blind alley" of pile-driving formulas—there is an open road for the designers of pile foundations.

Fundamentally, a pile foundation is essentially the same as other foundations—the piles merely by-passing weak strata and transmitting the load to a stratum at a lower depth, sufficiently firm to carry the overlying strata plus the weight of the new structure.

The design of a pile foundation often requires great skill and experience, and cannot be solved by the application of a single pile-driving formula. Piles are often used when they are of little use, merely because the designer was not sufficiently informed as to the developments of soil mechanics.

It should be clearly emphasized that reports *A* and *B* are divergent and opposed. Report *A* still expresses faith in pile-driving formulas and proposes one of its own—a modified Hiley formula. Report *B* expresses doubt on all pile-driving formulas and states (Paragraph *B*-12) that

> "Load testing of piles is the only reliable method for determining the load which a pile can safely carry in relation to the shearing strength of the soil surrounding the pile."

Report *B* states in Paragraph *B*-1 that,

> "It should always be kept in mind that pile formulas and pile tests cannot yield information on the magnitude, distribution, and time rate of the settlements for an entire structure."

Until about 1929, although engineers had kept records of settlements on structures and knew the general form of such settlements, there was no method of computing such settlements in advance of construction. This gap was filled by the application of the Boussinesq theory to obtain the vertical pressure in the soil. Such pressures, when plotted in isopressure lines, give the bulb of pressure "which has been checked experimentally in the use of pressure gauges and also by photo-elastic methods"; but the great step forward was the introduction of the consolidation apparatus by Karl Terzaghi, M. Am. Soc. C. E., to obtain the rate of compression and porosity of small undisturbed samples of the underlying soil, and to obtain the constants needed in his formula.

Whether to use piles or not for a foundation is often a difficult question to decide. There are many kinds of grounds unsuitable for piles (for instance, soft clay) in which the disturbance and remolding effect of the pile driving may be extremely detrimental and may hasten settlements. In general, piles may be effective in sands and the driving may consolidate loose sand—they may be destructive in silts and clays.

The question of whether to use a pile foundation can be answered only after a thorough study of the underground by borings, by tests on soil samples and a knowledge of the past history or geology of the site, which is a first-class engineering problem dealing with complex natural conditions. This is a problem for engineers and not one to be solved by a simple substitution in a formula after the various coefficients are determined or guessed at.

To illustrate the nature of the problem of pile foundations, the conditions along the Atlantic Coast may be cited. As is well known, this coast is now depressed relative to mean sea level. During glacial times the volume of ice was represented by about 50 to 100 ft of water in the North Atlantic. This is borne out by the presence of old sea beaches at this horizon. In places this beach is a "marl" of sand, clay, and shells, 30 to 40 ft thick. In places the "marl" layer is overlain by silt and underlain by soft clay; in other places underlain by old cretaceous sands and other hard deposits. In the case of a structure whose footings reach the "marl" it would be worse than useless to drive piles through the "marl" into the soft clay below. All that can be done is to utilize the marl layer to distribute the load to the clay below. What happens depends upon the compression of the clay, which is a general problem in soil mechanics.

In other cases, where the old solid formations are within reach, piles can be driven through the silt to hard bottom affording a good support and by-passing the silt so that it is no factor in the problem. This is a good use for piles.

Another case where piles are often used and proved to be ineffective is when a fill of unconsolidated sand or other material is deposited over a layer of mud or silt in some bay or river. At first, where piles are used, they seem to be effective; but as the fill settles and grips the piles a great load is thrown on them and the entire fill, with piles and structure above, settles. In this case, unless a temporary use is to be made of the structure or the piles, it would be better to provide a foundation such that the columns can be jacked up from time to time and fillers placed between the base of the columns and the supporting footings.

Various reports and papers of the Society have thrown considerable light on the subject of pile foundations and settlements of structures, such as the 1932 Progress Report of the Committee on Earths and Foundations[24] in which a comparison was made between the actual settlements and the theoretical settlements of a large structure on piles.

In a comprehensive study of the subject of dynamic pile-driving formulas, A. E. Cummings,[25] M. Am. Soc. C. E., concluded that "The installation of a satisfactory pile foundation is largely a matter of experience and good judgment combined with a careful soil investigation." Mr. Cummings does not believe that Hiley's formula as proposed in Report *A* is mathematically correct.

[24] *Proceedings*, Am. Soc. C. E., May, 1933, p. 777; see also "Settlement Studies of Structures in Egypt," by Gregory P. Tschebotareff, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 919; and "Practical Application of Soil Mechanics (A Symposium): Settlement of Structures in Europe and Methods of Observations," by Charles Terzaghi, *Transactions*, Am. Soc. C. E., Vol. 102 (1938), p. 1432.

[25] *Journal*, Boston Soc. of Civ. Engrs., January, 1940.

The writer hopes that the formula favored in Report *A* will only be considered as useful to encourage discussion and will never be adopted by the Committee or by the Society.

JOHN G. MASON,[26] M. AM. SOC. C. E.[26a]—The Committee will fail of its purpose if it adopts, in its present form, Report *B*, "Pile Formulas and Pile Tests." Pile-driving formulas are a necessity. If the Committee does not recommend one or more, the lay engineer will adopt one of his own and the present dilemma will still prevail. Paragraphs *B*-1 to *B*-9 of Report *B* comprise an excellent statement of the problem.

Paragraph *B*-10 emphasizes many objections to the various elements incorporated within proposed dynamic as well as static formulas, leaving the impression that such formulas are too complicated for general use, but at the same time suggesting even more complicated measures as a remedy.

Both dynamic and static formulas depend on soil analysis for the absolute security or safety of a foundation as a whole. This soil analysis or "soil mechanics" is indeed the primary consideration of the designer; the capacity of any one individual pile is purely secondary.

Paragraph *B*-10 does suggest rational determinations of the few variables that compose the functions in dynamic pile-driving formulas. However, the sum of all energy losses could be ascertained by lumping them in one field trial by finding the efficiency of the pile-driver rig. This could be done by comparing the load test to failure of a driven pile with the total energy developed by the blow. This method is a simple process. It will be shown, however, that there is a rather faithful consistency within each type or combination of pile-driving outfits tested and compared by field experiments.

It is generally acknowledged that the absolute energy of a falling body may be designated in foot-tons by the formula,

$$W\,h = E \qquad (39)$$

in which W represents the number of tons; h the number of feet (the body has fallen); and E the absolute energy. This becomes $12\,W\,h$ in inch-tons. When such a blow is delivered to a pile, the pile moves a distance, s, in inches, and at the same time offers a resistance, P, in tons. If ideal conditions existed and if no energy losses occurred in the falling hammer, during impact, or due to elastic deformation of either the hammer, cushion, or pile, and if no friction or heat loss was sustained by the pile movement under the blow, the two expressions could be equated to obtain (in tons) $12\,W\,h = P\,s$; or:

$$P = \frac{12\,W\,h}{s} \qquad (40)$$

Any pile-driving rig may be rated as to its efficiency e by performing a load test. When loaded to failure, the test load may be designated as T. Then the efficiency would be

$$e = \frac{T}{P} \qquad (41)$$

[26] Bridge Engr., State Dept. of Roads and Irrig., Lincoln, Nebr.

[26a] Received by the Secretary August 25, 1941.

Under the foregoing ideal conditions, Eq. 40 represents the theoretical ultimate capacity of the pile under a given blow, without regard to the efficiency of the driving operation or safety factor. Call the safety factor F_1, and the efficiency factor F_2. Then, the ultimate formula (Eq. 40) may be transformed into a working formula as follows (safe load in tons):

$$R = \frac{12\,W\,h}{s \times F_1 \times F_2} \qquad (42)$$

The safety factor, F_1, is arbitrary and should be made to vary in accordance with the importance of the work, but the factor F_2 will be fixed by actual field data.

It will be noted that Eq. 42 corresponds with Eq. 3 of Report A, the difference being that in the former the term F_2 in the denominator is substituted for the deduction factors expressed by the terms, $e\,W\,h\,\frac{P(1 - n^2)}{W + P}$ and $R\,d\,k$ in Report A.

Eq. 42 accounts for all energy losses, large and small, without bothering to identify and analyze each individual one of them. They are measured, summed, and lumped automatically in the field determination of e and F_2.

A few tests of this suggested method have been made by the compilation of actual field data assembled from various sources. The fifty-three record cases used herein were furnished and recorded from driving and loading tests made by the State of Nebraska under the direction of the writer, and from driving and loading tests made by, or recorded by, the Carnegie-Illinois Steel Corporation, observed and compiled by G. G. Greulich, Assoc. M. Am. Soc. C. E.

These tests have been divided into four groups as follows:

Group No. 1	Group No. 2	Group No. 3	Group No. 4
Steel piles	Steel piles	Timber piles	Solid concrete piles
Gravity hammer	Single-acting steam hammer	Single-acting steam hammer	Single-acting steam hammer

Table 10 shows the efficiency e, and the resulting factor F_2, and also F_1 which has been chosen arbitrarily with the constant factor of 3. Suggested values of R may be computed by:

$$R = \frac{4\,W\,h}{s\,F_2} \qquad (43)$$

with F_2 as given in Table 10.

Table 11 shows in detail the data for each individual test. Several other tests were omitted where:

(1) Piles were driven to refusal;
(2) The test load exceeded the theoretical ultimate capacity of the pile; and
(3) Driven piles were tested to satisfy specification requirements only, and which did not test the pile to failure.

It must be observed that where any data are used with nearly infinitesimal penetrations per blow, all laws of physics and common sense are being violated.

TABLE 10.—VALUES OF EFFICIENCY

Pile type	Hammer type	e	F_2	F_1	No. of tests	Tests made or recorded by
Group No. 1 (steel)	Drop hammer	32.55	3.07	3	16	State of Nebraska (writer)
Group No. 2 (steel)	Steam, single acting	44.4	2.25	3	21	19 by Carnegie Steel Corporation (Greulich) 2 by State of Nebraska
Group No. 3 (timber)	Steam, single acting	57.6	1.77	3	6	Carnegie Steel Corporation (Greulich)
Group No. 4 (concrete)	Steam, single acting	21.7	4.6	3	10	Carnegie Steel Corporation (Greulich)

TABLE 11.—PILE-DRIVING EFFICIENCIES

(a) DROP HAMMERS; STEEL PILES (NEBRASKA TESTS)[c]

No.[a]	W	h	s	P[b]	Test load	e
3[c]	1.02	10	0.92	133.0	32.7	24.5
4	1.02	10	1.64	73.5	24.0	32.6
7	1.00	26	0.874	358	60.0	16.7
8	1.00	12	0.6	240	80.8	33.6
9	1.00	12	0.5	287	98.1	34.1
10	1.00	26	0.775	400	108	27.0
15	0.925	16	1.17	152	17.3	11.35
16	0.925	16	2.6	68.3	23.1	33.8
17	1.02	10	0.675	182.0	55.8	30.7
21	1.06	10	4.4	28.7	19.5	67.9
22	1.06	10	3.0	42.0	21.0	50.0
23	1.00	10	1.95	61.5	22.5	36.5
24	1.00	12.5	1.8	83.2	27.5	33.0
25	1.00	17	0.89	229.0	47.0	20.5
26	1.21	10	2.2	66.0	36.5	55.3
28	0.92	10	1.46	75.0	17.5	23.3
Total						520.85
Average efficiency, $e\left(=\frac{520.85}{16}\right)$						32.55
$F_2\left(=\frac{100}{e}=\frac{100}{32.55}\right)$						3.07

(b) STEAM HAMMERS; STEEL PILES

No.[a]	W	h	s	P[b]	Test load	e
9	2.5	3	0.273	330	105	31.8
15	1.5	2.4	0.50	86.4	50	57.9
17	1.5	2.4	0.52	83.0	50	60.2
29[c]	1.5	2.42	0.60	72.5	27	37.5
30[c]	1.5	2.42	0.54	80.5	27	33.5
28a	2.5	3	0.11	820	225	27.4
28b	2.5	3	0.11	820	300	36.6
29a	2.5	3	0.11	820	200	24.4
29b	2.5	3	0.11	820	307	37.5
33	1.5	2.42	0.13	336	60	17.9
60	1.5	2.42	0.65	67	40	59.6
63	1.5	2.42	0.52	83.5	49	58.7
64c	2.5	3	0.63	143.0	50	35.0
64d	2.5	3	1.33	67.6	62.33	92.0
64g	2.5	3	0.71	127.0	68.46	53.6
64h	2.5	3	0.44	202.0	86.85	43.0
65	2.5	3	0.32	282	74.6	26.4
66	2.5	3	1.1	82	74	90.0
67	2.5	3	0.55	164	73.5	44.8
68	2.5	3	0.14	644	136.0	21.1
69	2.5	3	0.24	375	111.45	29.6
Total						888.5
Average efficiency, $e\left(=\frac{888.5}{21}\right)$						44.4
$F_2\left(=\frac{100}{e}=\frac{100}{44.4}\right)$						2.25

(c) STEAM HAMMERS; SOLID CONCRETE PILES

No.[a]	W	h	s	P[b]	Test load	e
4[d]	4.5	4	0.23	940	153	16.3
5[d]	4.5	4	0.35	618	182	29.4
12[d]	2.5	3	0.17	527	70	12.9
26[d]	4.65	3.25	0.544	334	117	35.0
5a	3.75	3.25	0.24	608	177.5	29.2
5b	3.75	3.25	0.24	608	128.5	21.2
70e	4.5	4	0.29	745	123.7	16.6
70f	4.5	4	0.26	830	142.0	17.1
70g	4.5	4	0.23	940	152.6	16.25
71d	4.5	4	0.35	615	142	23.1
Total						217.05
Average efficiency, $e\left(=\frac{217.05}{10}\right)$						21.7
$F_2\left(=\frac{100}{e}=\frac{100}{21.7}\right)$						4.6

(d) STEAM-HAMMER DRIVEN TIMBER PILES

No.[a]	W	h	s	P[b]	Test load	e
7	2.5	3	0.23	391	93	23.8
8	2.5	3	0.43	209	81	38.7
21	2.5	3	0.75	120	69	57.5
22	2.5	3	0.92	98	97	99.0
23	2.5	3	0.40	224	100	49.1
29	1.5	2.42	0.48	91	70	77.6
Total						345.7
Average efficiency, $e\left(=\frac{345.7}{6}\right)$						57.6
$F_2\left(=\frac{100}{e}=\frac{100}{57.6}\right)$						1.77

[a] Except where noted, tests are those of the Carnegie-Illinois Steel Corporation. [b] $P > \frac{12\,W\,h}{s}$ (see Eq. 40). [c] Nebraska tests; all Nebraska tests in Table 11(a). [d] Tests by Greulich.

The low efficiency of single-acting steam hammers on solid concrete piles illustrates the necessity of specifications governing the ratio of the hammer weight and the pile weight.

It is unfortunate that the writer has been unable to assemble a larger number of tests in these various groups, but this small showing is intended to illustrate a possible simple solution of this problem.

It is the writer's opinion that a nation-wide average for the factor F_2 used in Eq. 42 will yield results as accurate as, or even more accurate than, may be obtained from any formula using theoretical deduction terms designed to correct for energy losses due to elastic compression and impact.

It is conceivable that if some one agency, such as the Society, could be induced to act as a clearing house for collecting such data from over the entire nation, sufficient data could be assembled in a single construction season to warrant the selection of a set of efficiency factors, F_2, for immediate use during the following construction season. If this same agency should be retained for succeeding seasons, these factors could be modified from time to time in accordance with the dictates of the ever-increasing wealth of data.

The end would be the introduction of a formula for which there is a demand and yet one that is simple to handle in the field. It would be flexible for modification to suit various conditions of soil and types of piles and pile drivers. The factor F_1 may be varied by specification to suit the degree of refinement any job may warrant. The factor F_2 may also be varied by specifications in accordance with the results of actual load tests for the particular job, or to conform with the factor recommended by means of a nation-wide average.

CARLTON S. PROCTOR,[27] M. AM. SOC. C. E.[27a]—The publication of this Progress Report will be of inestimable value to the profession if it succeeds in arousing the full discussion, including the "actual data determined from pile driving, application of formulas, and load tests * * *" as requested in the last paragraph of the "Foreword." There can be no question that (see "Foreword") "the engineering profession as a whole has not kept pace with the most recent developments, * * *" and that "so many engineers are dependent on the older methods, that the introduction of new methods involves two steps: (1) Placing of information before the engineers for a discussion by the profession; and (2) as a result promoting self-education on the part of the profession as a whole * * *."

Any criticisms of this Report must lie not in what is stated, but rather in what is omitted, since the large immediate value of this Report is in its warning to designing engineers as to the fallacies of pile formulas and the weaknesses of pile tests. The Report might well have gone one step further to point out and warn the engineer as to the delimitations and even hazards of pile installations under many conditions where piles have been used in the past. It is to be hoped that the publication of this Report will serve to correct the "unhealthy" effects of some of the advertising and sales propaganda that has

[27] Cons. Engr. (Moran, Proctor, Freeman & Mueser), New York, N. Y.

[27a] Received by the Secretary August 27, 1941.

influenced sections of the engineering and architectural professions in recent years, and will bring a general realization of the fact that, in many cases, the wrong type of pile has been selected for a foundation, or that, on occasion, piles have been used where they could serve no useful function and where in some cases they have contributed to foundation failure.

Although the Report does state in Paragraph *B*-9 that "the driving of the pile often produces changes in the physical characteristics of the soil," the value of this Report would have been strengthened had it been emphasized that, in cohesive soils, the driving of the pile produces a remolding within the soil immediately adjacent to the pile, and a displacement of the soil in volume usually equal to the total volume of pile penetration. Where such pile foundations are designed as friction piles, they may frequently so disturb and remold a cohesive soil that its strength is reduced to a point where the supporting value would have been greater under spread footings of areas equal to that of the pile caps alone, had no piles been installed. Where such pile installations are designed to penetrate through deep strata of cohesive material to receive underlying support, the heaving of the soil produced by the upward displacement resulting from the pile driving frequently lifts previously driven piles destroying their supporting value partly or completely.

This Report should serve to clear much of the atmosphere of mystery that has surrounded the function of the pile support, and should show that the function of a pile intended for the support of structural loads is basically a simple one in that, when properly used, it must transmit the support of such structural load through weak soils to underlying stronger soils. Where supporting piles do not perform this simple function (as in the case of piles embedded for their full length in substantially uniform cohesive soils), they are improperly and, in some cases, dangerously used.

Usually, of far more importance than the selection of the pile-driving formula are the questions of (1) whether any piles should be used as opposed to spread footings, mats, piers, cylinders, etc.; and (2) the type of pile that should be adopted; that is, whether a displacement pile or an open ended pile; whether friction or point bearing; whether concrete, steel, or timber, etc. Engineering history is replete with cases of pile foundation failures chargeable to the use of the wrong type of pile or to the use of piles where another type of foundation support should have been designed.

George Paaswell,[28] M. Am. Soc. C. E.[28a]—The presentation of this Report, with its disappointing bilateral recommendations, emphasizes again the fundamental misconceptions of the rôle of the pile. The pile is not an ultimate foundation device, but merely an intermediate device to transmit a footing load to or through a given stratum. As a corrollary to this thought, the pile possesses no unique or intrinsic function but simply reflects, by its motion under load, the state of strain in the soil about and below it.

Soil mechanics has produced no novel analysis of the pile. It has given an

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[28] Secy. and Treas., Spencer & Ross, Inc., Detroit, Mich.

[28a] Received by the Secretary September 8, 1941.

analytic method of computing foundation movements when the method of loading the substratum is prescribed. Recognizing these fundamental concepts, it is absurd to expect that bearing capacities may be determined by adding individual pile capacities, whether such capacities are determined by dynamic or static formulas or even by field load tests. When one persists in the quest for a pile formula, one ignores or merely gives lip-service to the science of soil mechanics. Soil mechanics and the pile formula are essentially incompatible.

In the development of the "pre-test" pile, it is interesting to trace the futility of the pile formula and the sure rôle played by the science of soil mechanics in the proper placing of the pile. The introduction of a pile below an existing foundation by hydraulic jacking was an old art. A bearing capacity could be observed by reading the settlement against the loading. When engineering science stopped at this point, the pile was the ultimate foundation; and subsequent settlement, which was inevitable, was just one of those unfortunate occurrences that could be remedied by the introduction of more piles in future installations. When it was recognized that the pile was an intermediate device only and the loaded substratum the primary foundation, then a scientific method was developed for materially decreasing settlement by maintaining the stratum in a loaded condition. That a group of piles reflected only the behavior of the stressed subsoil followed naturally from the study of underpinning operations. The pile formula had no place in this study.

If the Committee had taken the position that the behavior of one pile in a given stratum could be scientifically correlated to the anticipated behavior under the complete foundation, there may have been some significance in the pile formula quest. However, if soil mechanics is used in rational fashion it is obvious that the study of the soil formation by recognized methods of sampling is superior to any "yardstick" formula.

If a proper soil exploration has been made and if an analysis has been made of the distribution of stresses and strains in the soil on the assumption of a given type of loading, what function can the pile formula serve? A dynamic formula involving the question of longitudinal impact between a moving weight, a pile unit, and the immediate and consequent stress distributions in the neighboring and remote soil taxes the power of mathematical analysis to give a rational answer. The usual recourse is to apply coefficients whose wide range of values vitiates the intricate mathematical reasoning. The coefficients derived from the fundamental properties of soil may more rationally be used in the study of the soil movements as given by soil mechanics.

The plea for a pile formula is frequently based on the requirements of field control. A simple formula gives a definite procedure to secure a "good" pile and when all piles have been placed in accordance with such procedure, one has a conventional foundation and obviously a satisfactory alibi when unexpected (?) settlement occurs.

It is time to become chary of formulas that require "ripeness of judgment" and "good experience" in their interpretation and application. One does not use "ripeness of judgment" or "good experience" in seeking the section modulus

of a beam to sustain a given type of loading under specified supports. One uses engineering science. Engineering science should also be used in the design of pile foundations.

ABRAHAM WOOLF,[29] ASSOC. M. AM. SOC. C. E.[29a]—It is not the intention of this discussion to provide any data or information substantiating pile-driving formulas. The writer's experience in pile driving, from a design and field inspection point of view, began when Professor Terzaghi, in 1925, gave his soil mechanics lectures discussing the validity of pile-driving formulas. It was then emphasized that one should be careful with formulas, particularly the famous "Engineering News" formula. Of course, this left the young engineer in quite a predicament as to what to do without a formula. The writer has continued his practice of foundation engineering under this condition, and he believes that many other engineers feel this limitation.

The Committee should be congratulated in its attempt to clear up a situation long wanting a solution. Report *B* definitely proposes that no formulas be used, but that load tests be resorted to. This proposal is a difficult one to accept, but basically it is sound and correct. The questions then arise: Shall piles be tested singly or in a group; shall a short pile or a long one be used for the test; or shall the engineer wait and watch the actual building being completed to serve as a test load? It is evident, therefore, that the load test as a substitute for a formula becomes a problem of extensive scope.

The beginning of the entire pile-driving controversy dates back to the time when the "Engineering News" formula first appeared. Previous to, and many years after, that time, only wood piles were driven. Then, 5 tons or 10 tons seemed to be heavy loads. Today, to place 300 tons on a test pile is a common occurrence; and the designing engineer is beginning to look for a better formula than the "Engineering News" formula to substantiate these high values, without undergoing the expense of running load tests. Without such a formula it appears to be difficult to convince the "old-timer" of the 5-ton and 10-ton school that 75 tons is not too unreasonable on some of the newer types of piles. The profession has accepted the "Engineering News" formula in the past to such extent that today it wants a new formula as simple and as effective; and there is no doubt that the original formula did serve its purpose.

In paragraphs *B*-12 to *B*-15, Report *B*, the Committee clearly outlines the load test procedure, but it does not emphasize that such a load is a dynamic test, and under certain conditions the settlement data observed in these tests are not criteria as applied to the actual building. In cases where friction forms the larger component of resistance in cohesive soils, it has been proved that load tests have given results that have not been too favorable regarding settlement in the actual building. A static test (a long-time test) no doubt appears to be out of the question, since the time element makes it impracticable. Some consideration to long-time tests should be given, and there is no doubt that a relationship between the static and dynamic tests will become evident.[30]

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[29] Structural Engr., Albert Kahn, Inc., Detroit, Mich.

[29a] Received by the Secretary September 15, 1941.

[30] Discussion by Abraham Woolf of paper entitled "The Structure of Clay and Its Importance in Foundation Engineering," by A. Casagrande, *Journal*, Boston Soc. of Civ. Engrs., October, 1932, p. 447.

In addition to load tests and prior to the actual letting of contract for construction, the writer has found the following procedure of extreme value in approaching the problem from a design point of view:

1. Obtain good core borings to rock or hardpan with a driving record of the core pipe by the boring contractor.
2. If piles are the solution to the foundation problem and the strata are variegated, drive a test pile of heavy-grade steel pipe type, with a closed end. In some cases, several piles scattered through the site are necessary, where the substratum varies and the plot is extensive. Keep an exact driving record of these piles.
3. Coordinate the information of items 1 and 2 and determine what type of pile should be used and to what stratum it shall be driven. To a person of experience, the core pipe and the test pile will guide him to estimate the bearing capacity of the pile.
4. When the information obtained in items 1, 2, and 3 is so indefinite that one is not able to determine a bearing capacity for design purposes, then a dynamic load test must be made and further study given to these test values, the type of pile, and the mechanics of the soil surrounding the pile.

In the "Foreword" of the Report, it is stated that: "Formerly the wood pile was about the only type in extensive use, and some of the original formulas were devised with that pile in mind." It is suggested, therefore, that a new paragraph should be written giving consideration to the following factors:

Friction Piles.—The action of a friction pile in cohesive (clays) and non-cohesive (sand) soils; the effect of driving such piles through clays causing breakdown in the clay structure and in turn causing increased settlement,[31] and the distribution of the load from the pile through the soil strata surrounding the pile.

Point Bearing Piles.—The resistance of point bearing piles on rock, in sands, and in clays; and the effect of blunt ends and pointed ends.

Friction and Point Bearing Piles.—A study of the relationship between the two types of resistance and their combined effect on the total resistance of the pile.

Long Piles.—The study of long piles as to their buckling resistances in different types of soils when such piles act as friction, point bearing, or a combination of both; the lateral support (long-time effect) developed by cohesive and non-cohesive soils; and a study of connections in composite piles made up of two different types of materials.

Short Piles.—The column action of short piles as compared to long piles.

Shape of Piles.—A study of various shapes of piles—round, octagonal, square, H, etc., as adapted to different types of soils when used for friction, point bearing, a combination of both, or for compaction.

With the study of the foregoing factors a better understanding will be had of the action of the many and varied types of piles on the market at present.

[31] "The Structure of Clay and Its Importance in Foundation Engineering," by Arthur Casagrande, *Journal*, Boston Soc. of Civ. Engrs., April, 1932, p. 168.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

COST OF PUBLIC SERVICES IN RESIDENTIAL AREAS

Discussion

BY DONALD M. BAKER, M. AM. SOC. C. E.

DONALD M. BAKER,[9] M. AM. SOC. C. E.[9a]—The problem of rehabilitating residential areas where housing has fallen below the standards for which it was originally developed is one that has attracted considerable attention during the two decades since 1920, and will achieve much greater importance in the future.

During past periods of relatively high rates of population increase in metropolitan areas, land area devoted to residential housing likewise increased comparably. New housing was provided primarily for residents in the high-income and upper middle-class income brackets, whose vacated dwellings were then occupied by those with lesser incomes. With continued growth, these in turn moved on, to be replaced by a succession of residents in progressively lowering economic levels. Ultimately, an area was occupied by those in the lowest income brackets unless, as sometimes happened, such housing was located adjacent to an expanding commercial district. Then commercial or light industrial uses would invade the area, occupying the former residential buildings, or replacing them with newer buildings specifically constructed for particular uses. Until about 1920, little attention was given to the progressive "blighting" of older residential districts. Such phenomena were accepted as a normal result of metropolitan growth. Owners of property in such areas maintained hopes that expansion of commercial or industrial districts would in time reach their property, and give it values based upon such uses.

Community planning, as it is now beginning to be practiced, based upon the scientific method of collecting, analyzing, and interpreting factual data relative to growth and decay, developed a realization that the progressive decay of such areas was an effect, the causes of which were subject to determination, and that in many instances remedial measures were possible. Without question, economic losses due to "blighting" are very great, not only to

NOTE.—This paper by F. Dodd McHugh, Esq., was published in June, 1941, *Proceedings*.

[9] Cons. Engr., Los Angeles, Calif.

[9a] Received by the Secretary August 18, 1941.

owners of property within "blighted" areas, but likewise to entire communities where such property exists, and to residents of such communities.

Owners of such property experience not only reduction of value of land, because of lowered rentals upon improvements, but likewise a loss of adequate return upon that part of the improvement—the foundation and the shell of the structure—whose physical life, if it is well arranged and soundly constructed, may range as high as one hundred or more years. Reserves for depreciation and obsolescence are seldom if ever set up in individually owned properties, and are more or less impracticable. Community losses include a reduction of taxable values and loss of a part of the public's investment in improvements and services. These usually have a useful physical life in excess of the life of the area served for any but the lowest type of residential housing or, in time, they develop excess capacity due to population loss in such area.

Loss to individuals includes an increase in taxes to support cost of improvements and services originally provided for the "blighted" area, which fails to carry its share of such cost because of lowered taxable value, and loss to merchants and property owners in business districts serving the "blighted" areas. There is also the expense and loss of time caused to those who, daily or frequently, must travel between outlying residential areas and the central business district a greater distance than they were formerly required to travel.

Movement into new housing is caused by both "pulling" and "pushing" forces. Improvement in financial circumstances, attraction of new surroundings and environment, and more modern conveniences all initiate the movement away from established housing for those who can afford it. A "vacuum," slight in amount, is created, which tends to "pull" into the area residents in lower economic levels who can afford to move up the scale somewhat. Soon environment and neighbors have changed to a point where residents who were not "pulled" out of the area are "pushed" out. The cycle is continuous, and may require many years to change the character of a district from a high to a low level. In general, however, the rate of obsolescence and blighting of a residential district varies directly with the rate of population increase in the community as a whole.

With a reasonably high percentage rate of population increase, the supply of persons required to fill up the vacuum created by the outward flow of residents from older districts usually exists all through the scale. The 1940 federal census brought home to many people, however, the fact that most metropolitan areas, as well as the nation as a whole, are fast approaching a static condition in regard to population, and that former rates of increase will not be maintained.

This condition will undoubtedly bring the problem of the "blighted" area into a far clearer focus. Unless American society becomes far more static than it has in the past, with barriers between those in various economic strata becoming far more inflexible, there will always be a large segment of the population who will yearn for, and also be able to achieve, better housing, surroundings and environment, modern conveniences, and everything else that goes with improved circumstances. The demand for new housing will con-

tinue; and the "blighted" areas will increase also, but the supply of residents for these latter areas will not be forthcoming as it has in the past, and decay and loss of values therein will be accelerated.

New housing, as now produced, is usually supplied around the periphery of present developed areas. It entails the expense of the many facilities indicated by the author. These expenses are usually a community obligation. Arresting decentralization of population through rehabilitation of areas now in this condition, or approaching it, means large annual savings in dollars and cents to the community, and therefore to its taxpayers.

There is another side of this problem of rehabilitation, however, which pertains to the owner of property in a "blighted" area, who might desire to undertake rehabilitation, or to the investor who might desire to finance it. In many instances, rise in price levels over a long period, and likewise rise in values due to general increase in population of most communities, have reduced losses due to depreciation or obsolescence when property has changed hands. The temptation to spend all revenues received from rentals, except that immediately necessary for routine expenses, maintenance, taxes, interest, and amortization of loans, is increased by the feeling of rising valuations offsetting, at least to a degree, depreciation and obsolescence.

An owner, or an agency contemplating development of new housing to replace old existing housing in a blighted area, is faced with the following problems:

(1) A large area must be acquired if a proper environment is to be created, such area ranging from at least 40 acres to possibly several times this extent. Demolition of a few existing structures, and their replacement by new and modern buildings, will not create environment.

(2) To secure control of an adequate area, many individual properties must be assembled and purchased, or condemned, entailing large outlay of capital for land alone.

(3) When the cost of buildings is added to the capital investment required for land alone (which in a project of this nature would include land and obsolete improvements), any such project takes on very large proportions.

(4) When such investments are constructed upon unimproved land in peripheral localities, the owner's investment usually includes the cost of an unimproved site, of buildings, a certain amount of landscaping, and those public improvements and services not usually provided by the community or existing public utilities. It is questionable in many cases if the increased rentals received from new developments in a rehabilitated area will support the added site cost in such an area.

For years sociologists have stressed the economic losses resulting from substandard housing—losses resulting from sickness, from crime developed in such housing, and from increased costs of fire and police protection. These losses have even been evaluated in terms of dollars and cents. So far, research has failed to develop a type of low-cost housing within the reach of those in the lowest economic circumstances, without some form of subsidization. The propriety of public subsidization of housing for this part of the population has

recently been accepted, and many low-cost housing projects have been constructed or are contemplated.

With the definite community savings as shown by the author, which are affected by rehabilitation of "blighted" housing through site planning, and construction in accordance with new and modern standards, the writer poses this question: "Would it not be a sound investment if such projects likewise were to receive public subsidization from the community, at least to an extent which allowed the increased burden of added site cost to be overcome?"

The paper is a distinct contribution to the literature of this subject, made in a language that businessmen understand—that of dollars and cents.

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DISCUSSIONS

AN INVESTIGATION OF PLATE GIRDER WEB SPLICES

Discussion

By Francis L. Castleman, Jr., Assoc. M. Am. Soc. C. E.

Francis L. Castleman, Jr.,[7] Assoc. M. Am. Soc. C. E.[7a]—The designing engineer will benefit greatly from this series of investigations. As stated in the "Synopsis," the theory of the design of plate girder web splices has been based largely on assumptions that have never been checked experimentally.

Considered broadly, these investigations give confidence in the four types of splices investigated, and designers need feel no hesitation in using any one of them. A theory does not necessarily have to be correct; all it has to do is "work"! Even casual inspection of the results of these tests shows great departure from theory, but such results that the conventional methods are quite safe and will suffice until better and more rational ones are established and accepted. Certain conclusions can be drawn from these investigations that possibly can be interpreted as suggesting minor modifications in the present theory as used.

It is difficult to choose the optimum of the four splices tested, taking into account both theoretical and practical considerations. From the latter standpoint splices G_1 and G_2 are undoubtedly the best, having fewer pieces and being more simply fabricated. Their variation in strength from the other types is negligible when considered from a practical viewpoint. They also possess a certain theoretical merit in that on lines A and C, Figs. 5 and 6, a nearer approach to straight-line theory is shown.

From theoretical considerations alone, splice G_4 probably would be preferred by most designers. However, Table 3 shows this to be located on a girder that failed under the smallest loading. In other words, assuming that the splices influenced the failure of the compression flanges, this type of splice appears to have exerted the greatest damaging influence. To the writer this is one of the most interesting features of this series of investigations. It is

Note.—This paper by J. M. Garrelts, Assoc. M. Am. Soc. C. E., and I. E. Madsen, Jun. Am. Soc. C. E., was published in June, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: September, 1941, by Messrs. C. H. Gronquist, Charles Stratton Davis, and Bruce Johnston.

[7] Associate Prof., Structural Eng., Vanderbilt Univ., Nashville, Tenn.

[7a] Received by the Secretary September 15, 1941.

true that certain modifications could perhaps be made in this splice that would alter these results, but these modifications are by no means apparent upon starting the initial design. The designer has little time to ponder over such modifications even if they were finally forthcoming.

Splice G_3, which is the least rational of the four types considered, shows the greatest breaking strength. This is interesting, particularly to the writer, in that he has seen this type frequently criticized (as well as most other types) as lacking certain theoretical qualifications. It would appear that the ordinary methods used in calculating the strength of such a splice are of such a fortuitous nature that they inadvertently lead to great strength.

In designs of type G_1 and G_2 it is often common to reduce the working value of the extreme rivet in the splice plate by the ratio of its distance from the neutral axis to the distance of the extreme rivet in the web (through the flange angle) from the neutral axis. Since the tests under consideration show that each component part of such a splice carries only the stress in that part of the web plate beneath it, this would seem unnecessary. Whether desirable or not, the flange acts as a separate unit in splicing the cut web at the top and bottom, and its action is entirely independent of the splice plates on the web. It is interesting to observe here that in addition to the increase in flange stress set up by the splicing action this increase also throws an extra increment on the flange rivets in the vicinity of the splice.

From a standpoint of stiffness, which is often an important consideration, most splices are usually too short. Splices of the type of G_3 will overcome this as the tests show, but the same results can be obtained by increasing the length of splices of the type G_1 or G_2. In most cases this would mean extra material, and rivets would have to be added in the interest of increased stiffness. There is no valid objection to this, if it is desirable, as the same problem frequently arises in the design of main members.

In addition to the action of the splices these investigations give a very good picture of the buckling action of the compression flanges of the girders. Using G_3 as a basis, its theoretical modulus of rupture, by proportion, would be $\frac{17.6}{32} \times 65.3 = 36$ kips per sq in. This, in itself, is not sufficient to cause failure of the compression flange, but, when the increase in stress due to sidewise buckling, plus a small allowance produced by the splicing action, is added, it is easy to understand how failure occurred. It should be noted that the yield point of the flange angles is approximately 43 kips per sq in. and that of the $\frac{3}{16}$-in. cover 31 kips per sq in. Both of these values would have to be considered, as well as the restraining influence of the web, in determining the elastic instability of the flange.

It should be borne in mind that the four girders tested to failure had top flanges that were not supported laterally, or, if so, only that small amount furnished by the loading apparatus. Furthermore, the top ends were not restrained against sidewise rotation. Such an arrangement would scarcely be met in problems of a practical nature and therefore the results can be looked upon as extreme.

With regard to shear it is unfortunate that other loading arrangements were not investigated in addition to the symmetrical arrangement used. In the elastic range considered, shearing values obtained were so small that it is questionable whether a conclusive statement could be made safely concerning the lack of shear resistance at the horizontal planes where the splice plates were discontinuous. It is often necessary to locate a splice at a point of relatively high shear, and it is in such cases that these points of discontinuity become critical. In the cases under consideration, taking G_3 as a basis and using the breaking load, the theoretical horizontal shear at the vertical toe of the flange angle in the web, using the well-known relation $\frac{V\,Q}{I\,t}$, would be 6.4 kips per sq in. at splice 3. It is also interesting to observe that, at this same point and for the same load, the theoretical stresses on a cross section due to moment fall within the elastic range (but, of course, greater than the allowable).

It would have been instructive if at least one of the girders could have been loaded in such a manner that splice 2 would have been subjected to a high shear as well as moment. As given, no shear occurs at this splice, and the loading producing such a condition is generally only of academic interest, rarely, if ever, occurring in practice. In fact, the authors state under "Discussion of Results": "* * * there can be no failure of the splice due to moment until after one of the flanges has yielded." Of course, when this has happened the girder has failed structurally.

The ideal web splice would be welded, without the use of side plates, and with the webs butted together for a V-weld or a modification of it. With such an arrangement, and using coated electrodes, stress conditions would be practically the same as in the adjacent unspliced parts of the web. As yet there is no ideal riveted splice in the sense that a stress condition is set up similar to that in the unspliced web. The writer questions seriously if such a riveted splice, or a rational solution for it, can be devised. The rivets create so many individual load points that the consequent complicating ramifications become enormous and defy rigid theoretical analysis. However, experience, and the investigations under discussion, have shown ably that the present methods are adequate, even if they are not based on entirely sound theory.

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DISCUSSIONS

LAND SURVEYS AND TITLES

SECOND PROGRESS REPORT OF THE JOINT COMMITTEE OF THE REAL PROPERTY DIVISION, AMERICAN BAR ASSOCIATION AND THE SURVEYING AND MAPPING DIVISION, AMERICAN SOCIETY OF CIVIL ENGINEERS

Discussion

By Dorr Viele, Esq.

Dorr Viele,[3] Esq.[3a]—Apart from the content of this Second Progress Report, there is intrinsic interest in the background of the study of which this interim report is an expression. A great impetus toward realizing potential improvement in the records of land surveys and titles is found in gaining acquaintance with the country-wide net of fixed points established by the U. S. Coast and Geodetic Survey and with the Survey's system for utilizing those points in identifying tracts (even small ones in private ownership), known as the state plane coordinate system.

The Committee's sessions to date (about nine since October, 1937) have given the legal members more familiarity with the engineering concepts involved than is gained only from the papers printed in *Proceedings*.[1, 4]

There is an interrelation between land surveying by the civil engineer and conveyancing by the lawyer, because to describe land parcels is an essential object for each, toward which they can cooperate reciprocally; but past developments show that recognition of such supplementation was hindered by the slow advance of surveying technique in the face of physical conditions in settling various areas of the present United States, while within, say, the forty years since the beginning of the century, surveying has advanced far ahead and conveyancing skill appears to be declining.

Note.—This Report was published in June, 1941, *Proceedings*.

[3] Counsellor at Law, Boston, Mass.

[3a] Received by the Secretary September 10, 1941.

[1] *Proceedings*, Am. Soc. C. E., November, 1938, p. 1879.

[4] *Ibid.*, April, 1939, p. 576.

The early factor that allowed verbal land description to spread independently of surveys was the chance step of requiring deeds to be publicly recorded. Enforcing this requirement (which is really the result of efforts aimed at something else) has ever since concealed a general absence of public regulation of plats expressing the findings of surveys. The facts may be learned advantageously by both professions through elementary analysis and history.

A suburban landowner desiring to create a public knowledge of what his possession embraces cannot usually get his mayor and fellow citizens to accompany him in a walk around his boundary that he may show them its lines. Even should they be ready to go with him, it might happen that he could not convincingly identify the exact limits of his acreage or point out anything to cause them to be remembered later. This is true at least in most of America. The English "beating the bounds" arose as a practical custom for impressing on memory the lines and corners of unchanging units of ground that were seldom transferred and could be as well identified by name or nickname as an owner himself. Yet either a direct demonstration of bounds or a designation of a property by its name presumes continuance of a pre-existing knowledge—of location, size, and marks—and has done nothing to reduce the knowledge to a form more convenient for transmission or preservation. In American custom (whether as cause or result) that convenience can be achieved only by a representative description of the parcel by words or a plan.

At the settlement of America's eastern seaboard the art of word description was exercised with almost as much difficulty as the newer art of cartography. Applying either art to landholdings then was novel from the point of view of England, where a property kept its familiar shape and name, was infrequently transferred, and if sold was handed over in the presence of witnesses, by "livery of seizin," through the symbol of a turf or twig or perhaps a door key, and there were no deed registries. Possession was the great element of title and its ceremonious delivery at the parcel supplied the publicity of the rightful title in the new owner. While thus in old England conditions coincided to continue the sufficiency of long custom, earliest New England discovered an abrupt absence of all of them. There were no old holdings familiarly possessed, no walled parcels of any kind, nor man-made permanent marks to grow into the knowledge of successive generations. When the first big fleet came with John Winthrop to Massachusetts Bay, for purposes of white settlement the country between Plymouth and Salem was as it had been before Columbus. A company numerous enough and foresightedly prepared then, for the first and last time in Anglo-Saxon annals, brought its living ways to a new wilderness.

Before the sailing, the Massachusetts Bay Company had bound itself to distribute land to the stock subscribers, 200 acres per fifty pounds subscribed, and by deed if demanded; but it wasn't. The voyagers wanted to go ashore. If, in the transferring of hundreds of men and families from shipboard to their lands, the Governor and Council had wished to delay to issue deeds, what descriptions would identify lots intended? In fact, the leaders imposed a strict control for the general benefit. Locations were selected by and for groups or "towns," with the bounds between drawn as generously as possible, and these town bounds were thereafter to be perambulated at intervals by a group from

both sides of the line. Within them, the "lots" for dwellings—near the area's center, and all of modest size—were equivalent each to each.

A list of the householders participating in the "first division" started the town's record or corporate minutes wherein note was made when any one was later voted a lot, and in time there was occasionally noted some one's buying from a previous holder his "house lot and his share of meadow." It was sufficient description while the lots and shares were a standard size.

Not until 1634 was a general listing sought of the landholdings of each voter (freeman). It was directed to be made by each town choosing four or more leading men to join with the constable; the listing, certified to the court and entered in its record, was to be proof of the possessor's title. The next sixteen years were used toward that design, which has never been completed. The story of the establishment, by 1652, of the system of recording deeds—indigenous to America and due to its crude conditions for the seating of an organized society—is found in Massachusetts colony archives that the writer has examined and transcribed, having lacked the opportunity before his analysis of the system's results.[5] Records so old as to be of themselves useless for their original purpose, because time has destroyed the recited evidences they relied on, have still a value in disclosing their scope and methods.

The early small-parcel land descriptions recite as identifying factors contemporary facts of possession and purchase, with the simplest indicators of location and perhaps size: "All that their house and ground in Dedham" (1640); "All those his lands at Mt. Wooliston containing 112 acres more or less which of late he bought of Thomas Gornell of Boston" (1640); "* * * his house yard and garden in Boxford with the privileges and appurtenances" (1651). They may use adjacent owners and natural features: "One lot of land lying at Stony River in Roxbury Towneship by estimation 20 acres of upland and marsh or thereabout and lying between the lands of Mr. Thomas Weld and Widdow Lambe" (1652); "Two acres of meadow in Fowl Meadow formerly purchased of Capt. Atherton being bounded with John Frairy on the South, a small river on the West, Peter Woodward on the North and a hilly small island toward the East" (1646); or a well-known building: "fiveteen acres of soult marsh medow lying by Dorchester Tidemill the lands of Thos Robinson lying on the Northerly side" (1652).

If surprise is felt at the omission of a practical, customary connection between verbal and graphic descriptions, it is to be remembered that this early Massachusetts area was settled by the possessors of the charter, acting directly for themselves, in contrast to the later subdividing from a central office in the "proprietary colonies." In New Jersey, for instance, the proprietary minutes of extensive tracts are still preserved in the surveyor general's offices for East and West New Jersey at Perth Amboy and Burlington.

In the nine towns that by September, 1630, were settled, named, and taxed, one's name in the town book's list of citizens was sufficient proof of owning land. Early marking of town-bound lines by initialed trees habituated a use out in the country, where some chose to take their large stock subscribers' allotments, of blazed trees for private bounds without too nice attention to

[5] *Political Science Quarterly*, Vol. XLIV, No. 3, September, 1929.

selecting the trees. The earlier the occupancy of the "country" grant, the less likelihood there would be of any conflict with its location. Surveys according to the skill available were presupposed, and, before long, orders making outlying grants also named the two or more men to go (as the public's agents) and locate the area granted and subscribe a plat of it. The rule of thumb was to avoid conflicts in possession; and in the absence of all reference points, other than the casual marks placed, records of mutual acquiescence were valued but did not extend to recording plats with the deed records.

Two illustrations relate to town lines:[6]

> "The 27th of the First Month 1643: At a meeting by us whose names are underwritten, chosen by the towns of Salem & Ipswich, & having full power either town to agree & determine of the bounds between the said towns, do in the behalf of each towne agree & determine the same as followeth:
>
> 'Imprimis, we conceive that the meeting-houses of the two towns stand from each other NNE half a point Easterly, & SSW half a point Westerly; Whether it bee exactly so or no, we are fully agreed that the line betwixt the two towns shall run as followeth, viz: From the bound tree near John Fairfeilds house WNW half a point Northerly, and ESE half a point Southerly, as the trees are marked both ways from the said bound tree.' "

The signatures of Roger Conant and seven others follow.

From a partition agreement made between "Wooburne & Linn Village now called Redding"[7]—

> "* * * the line between them shall begin at the little brook in Parly meddow, where it begins to turn upward toward the NE, & so to abut upon Charlston head line, & to run N & by W into the country; the point being taken from the needle, without alowance for the variation, unto which agreement we have set our hands this 23 3d mo. 1644 Geo. Cooke John Oliver appointed by the Court to settle the bounds."

In 1682 it was ordered that thereafter grants were to be laid out or surveyed only by surveyors appointed by the court; yet the habit of recording deeds (the recorder being paid to record what the parties brought), or their own indefiniteness, veiled the value to the public of further correlating data of these surveys. They were filed with the General Court and are still preserved in the state archives, good delineations of vari-sized parcels but without latitude and longitude or other permanent identification of location.

The modern way of surveying may be said to have come here with the Ellicott family. The second Joseph reported his running an

> "Astronomical Meridian line with a Transit Instrument in which nothing was left to the uncertainty of the Compass. The part of the Country being almost everywhere covered with a thick heavy growth of timber, to run this line required an immensity of labor to effect. Its whole length from Pennsylvania to Lake Ontario (92 miles 27 chains and 16 links) required from 1st July to 28th October 1799."[8]

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6 *Massachusetts Colonial Records*, Vol. II, p. 35.

7 *Ibid.*, p. 75.

8 "Holland Land Company's Papers," The Buffalo Historical Soc., XXXII, 1937, Vol. I, p. 49; compare "Pioneer History of the Holland Purchase of Western New York," by O. Turner, 1849, p. 406 *et seq.*

About 1800 the bothersome doctrine of "marketable title" had lately originated in England.

The best perspective on the study that is being digested by this committee is furnished by the histories of the two professions (law and civil engineering) during the growth of the United States, still so new that original documents and source studies, on mapping especially, are being published increasingly of late. Although it is to be hoped that the reposeful type of old homestead in unbroken family descent (instanced today from the original grants in Salem and Ipswich, Mass.) will never be eliminated in the United States, the widest concern is for current transfers of particular land areas for specific present uses. Processes that will expedite such transfers, while making the closing both more certain and cheaper, are worth open-minded investigation.

The big mutual interest of both bodies lies in their recognizing how both professions have suffered from their earlier aloofness and resulting self-sufficiency—lawyers in the pains of developing maxims and precepts as guides to compensate for the want of exactly fixed and recoverable points for bounds, and in their unawareness of how such bounds can now be obtained; engineers in the number and variety of recorded legal instruments to be consulted, and trouble and expense in establishing location of their intended starting points; and both, from the long-continued use of forms and ways that were shaped by earlier crude conditions and are not tolerable by modern standards.

Frankly, it is not to be expected that the real-estate lawyer will learn the engineer's technique to render boundary disputes impossible by measuring distances in even a precisely defined "projection of a spheroid," but the writer believes that there are members of the American Bar Association capable of recognizing that that technique exists, and that when they know from whom they can obtain its results for their own use they will learn to call for them.

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DISCUSSIONS

DYNAMIC STRESS ANALYSIS OF RAILWAY BRIDGES

Discussion

By R. K. Bernhard, M. Am. Soc. C. E.

R. K. Bernhard,[16] M. Am. Soc. C. E. (by letter).[16a]—Mr. Chew's discussion certainly represents a welcome addition to the paper.

In order to simplify the setup of the nomographic charts, the writer tried deliberately to avoid more complicated formulas. The accumulative effect of unbalanced wheel loads of steam locomotives has been proved experimentally. Numerous stress-time diagrams as given, for example, by J. B. Hunley in 1935,[5] or as early as 1928 in the report of the Bridge Stress Committee,[7] give sufficient evidence.

It would be rather simple to change the nomographic charts slightly in order to adapt them for transverse torsional vibrations. This case corresponds to self-sustained vibration due to aerodynamic instability of suspension bridges excited by horizontal wind loads.[17]

It must be emphasized, however, that investigations on actual bridges to determine the damping values more accurately are essential.

Note.—This paper by R. K. Bernhard, M. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1941, by R. S. Chew, M. Am. Soc. C. E.

[16] Prof. and Head, Dept. of Eng. Mechanics, The Pennsylvania State College, State College, Pa.

[16a] Received by the Secretary August 18, 1941.

[5] "Impact in Steel Railway Bridges of Simple Span," by J. B. Hunley, A.R.E.A., Vol. 37, No. 380, October, 1935.

[7] "Report of the Bridge Stress Committee," published under the authority of His Majesty's Stationery Office, London, 1928.

[17] "The Failure of the Tacoma Narrows Bridge," report of board of engineers, O. A. Ammann, T. von Karman, and G. B. Woodruff, March 28, 1941.

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DISCUSSIONS

WIND STRESS ANALYSIS BY THE *K*-PERCENTAGE METHOD

Discussion

BY MESSRS. FRANCIS L. CASTLEMAN, JR., AND CLYDE T. MORRIS

FRANCIS L. CASTLEMAN, JR.,[14] ASSOC. M. AM. SOC. C. E.[14a]—For the mathematically minded, or those who insist on extreme refinement in the accuracy of the assumptions and calculations, the *K*-percentage method of wind stress analysis will offer little appeal. To practicing engineers who, on occasion, must utilize the conventional methods, either theoretical or approximate, the *K*-percentage method and the accompanying new method of cantilever design offer a means whereby wind stresses can be determined with an accuracy approaching the so-called theoretical methods, and with little more time than that required for the simplest of the approximate methods. Verification of the accuracy of these new methods has been backed up by sufficient research, both analytical and experimental, to warrant full confidence in their use as presented.

The studies evolving these new methods, although very circumspect in the cases and conditions considered, lead, upon thoughtful consideration, to questions definitely not considered by them, but to which they are unavoidably joined and for which additional information is needed. The method of *K*-percentages can be used on any tier building, as can the new cantilever theory; but, as inferred in the paper under "Present Methods of Design," the conventional approximate methods are satisfactory for buildings of the usual proportions. Inspection of the building codes of various cities, however, leaves one in a quandary as to what a building of the usual proportions is.[15] Lacking any conclusive information, analytical or experimental, the following three cases are proposed arbitrarily:

NOTE.—This paper by F. P. Witmer, M. Am. Soc. C. E., was published in June, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1941, by C. M. Goodrich, M. Am. Soc. C. E.

[14] Associate Prof., Structural Eng., Vanderbilt Univ., Nashville, Tenn.

[14a] Received by the Secretary August 11, 1941.

[15] *Carnegie Pocket Companion*, 24th Ed., p. 357.

$$\text{Case 1; } \frac{\text{Height}}{\text{Least width}} \leqq 2\text{; wind stresses neglected.}$$

$$\text{Case 2; } \frac{\text{Height}}{\text{Least width}} \leqq 5\text{; wind stresses to be considered.}$$

$$\text{Case 3; } \frac{\text{Height}}{\text{Least width}} > 5\text{; tower buildings; wind stresses to be carefully determined.}$$

Of course, it is assumed in cases 1 to 3, inclusive, that the same intensity of wind pressure exists. Modifications should naturally be made for allowable increases in unit stresses (*a*) where wind is involved (an almost universal specification) and (*b*) where wind is omitted over protected heights.

Cases 1 and 3 offer no difficulty. In case 1, assuming the cases divided correctly, no consideration need be given; in case 3 wind stresses must be determined carefully either by one of the theoretical methods or by the method of K-percentages, preferably based on the new cantilever method. For extreme upper limits a combination of methods may be preferable before final sizes are determined.

Case 2 is the range in which most buildings that require wind stress analysis occur. Offhand, it would be assumed that all buildings in this class can be designed by any one of the approximate conventional methods (probably either the portal or cantilever method). The writer has often questioned this and felt that in the upper limits of this range, say from 4 to 5, more careful consideration should be given, but that the great accuracy (somewhat imaginary) furnished by the theoretical methods was not necessary. The method of K-percentages here offers a method that is precisely suitable and, in this range (or an equivalent one), will find its maximum use.

A series of studies, either analytical, experimental, or both, to determine ranges similar (but not necessarily identical) to those given by cases 1, 2, and 3, would be most helpful to the designer, who could then at once classify his building and the method to be used in its design for wind. It would seem that this series of studies should be made by methods similar to those that evolved the method of K-percentages and was a logical extension of the same.

The author states under "Analysis of a Compromise 3-Story Bent" that "* * * a design based on the assumption of a cantilever relation between vertical wind reactions may not be the most economical as to weight although it is to be recommended * * *." This would appear contradictory and at least only partly true. Obviously, if used, the cantilever method of design will fall under case 3 and then should be used when the secondary moments from column shortening cause the sizes of the members to be in excess of those obtained by the new cantilever method. Secondary moments from column shortening are not objectionable provided they are cared for and that the sum total weight involved is less than that required by cantilever theory. Again it is suggested that a series of supplementary theoretical studies should be made determining an approximate height-to-least-width ratio where this change takes place. Such studies might in themselves lead to shorter methods for determining the secondary moments set up in the beams from column shortening.

The bent as designed in the paper offers several interesting possibilities for variation in design, subject to possible verification by future analytical and experimental investigation. Two cases will be considered, both dealing with the girders of the first floor. (A similar procedure can be followed for girders of the other floors if desired.) All sizes are based on gross section following a similar procedure used in the paper. Inside girder G_1, as designed, is a 36-in., 160-lb H ($I = 9{,}738.8$) and was determined from the fact that its moment of inertia must be at least 4.70 times that of outside girder G_O. For vertical load alone a 27-in., 91-lb H ($I = 3{,}129.2$) is sufficient. Suppose cover plates are added to this at each end to provide enough strength to furnish a moment of inertia of at least 9,560 in.^{4} Calculation shows that a 27-in., 91-lb H gives poor size covers and that the base section is changed to a 30-in., 108-lb H with covers $16 \times \frac{11}{16} \times 6.75$ ft top and bottom at each end. The point of contraflexure for combined loading is taken as three eighths the span of 30 ft, and 1 ft is added to the theoretical length of each cover. For the end portions $I = 9{,}602$ in.^{4} and for the center portion $I = 4{,}461$ in.^{4} The end portion comprises 22.5% of the 30-ft span. It should be noted that this girder should be connected to the column for its moment-resisting capacity rather than the total end moment of 8,025 kip-in. For shear it should be connected for its moment-resisting capacity divided by one half the span. With the foregoing arrangement the ratio of the end moment of inertia of the inside span to that of the outside span is 4.72. A weight saving of 11.4% is obtained. For total economy this saving would have to be balanced against the increased cost of fabrication. Obviously, the K-values have been upset and only by future additional studies could such a procedure be justified theoretically. However, it should be noted that the author has used cutoff cover plates on the first-story columns. In all fairness it should be stated that such a procedure has much less influence on the design than that of using cutoff covers on the girders.

A more rational method than the foregoing, and one conducive to greater economy, is as follows: Replace the author's design of the outer first-floor girder G_O by an 18-in., 55-lb H ($I = 889.9$) with covers $10 \times \frac{3}{8} \times 3.75$ ft top and bottom at each end. This gives an I-value in the end portions of 1,531 in.^{4} and in the center portion of 889.9 in.^{4} The point of contraflexure for combined loading is taken at three eighths the span of 20 ft, and 1 ft is added to the theoretical length of each cover. Sufficient resisting moment is furnished at all points. The end portion comprises 18.75% of the total span of 20 ft. A weight saving of 12.7% is obtained.

From the foregoing design multiply the I-values in the end and center portions of outside girder G_O by 4.70, giving theoretical values of $I = 7{,}200$ in.^{4} and 4,190 in.^{4}, respectively, to be used in the design of the inside girder G_1. Again, for the same reason, a 30-in., 108-lb H ($I = 4{,}461.0$) is used for the base section. Covers 5 ft long, top and bottom, are used at each end with the point of contraflexure for combined loading taken at three eighths the span of 30 ft. One foot is added to the theoretical length of each cover. Sufficient resisting moment is furnished at all points with the added feature that, in the end portions, $I = 7{,}221$ in.^{4} and in the center portion $I = 4{,}461.0$ in.^{4} The

ratio of these I-values, respectively, to those in the side span is, for the end portion, 4.71; and, for the center portion, 5.01. These do not vary greatly from the theoretical value of 4.70. A weight saving of 24% in the inside bay is effected by this arrangement. The end portion comprises 16.70% of the total span of 30 ft.

The latter case, in which girders in the inside and outside bays are both cover-plated and the ratio of the moment of inertia is held to the value of 4.70 as nearly as possible, is more effective than the case in which cover plates were used only in the inside bay. That this is true is seen by the much greater saving in weight. However, it must again be emphasized that such a saving can be justified only by the verification of more extended investigations both analytical and experimental.

Professor Witmer's studies in wind stress analysis have opened up a new approach to this interesting field. Based upon both analytical and experimental evidence that is incontrovertible, they offer to the profession two new approaches that considerably simplify a somewhat difficult problem. As stated in this discussion the investigations can be extended considerably further and the writer sincerely hopes means can be found whereby this will be done. An interesting possibility would be an adaptation or modification to the Vierendeel truss. A tier building is no more than a cantilever Vierendeel truss.

Clyde T. Morris,[16] M. Am. Soc. C. E.[16a]—The design procedure outlined by Professor Witmer gives results closely in accord with the Spurr method of design.[3] The fundamental relationship on which the K-percentage method is based ("that I_g varies as $V_g\, L^2_g$"), so far as the writer is aware, was first called to the attention of the profession and utilized as a basis for design in a paper, read at the meeting of the Structural Division of the Society on October 13, 1927, entitled "The Design of Tall Building Frames to Resist Wind,"[17] by the writer in collaboration with A. Ward Ross, Jr. Instead of the K-percentage method as developed by Professor Witmer, the writer's paper presented a chart showing the variations in girder shears due to changing ratios of column-to-girder K's.

In discussing Table 3 Professor Witmer states that when the horizontal deflection of the building exceeds a proper assigned proportion of its height, it may be corrected by making a proportional reduction in the allowed column unit stress, keeping the stiffness relation unchanged between outer and inner columns. According to Professor Witmer: "A change in girder stiffness will have little effect upon deflection." This is in direct contradiction to the statement made earlier, in his discussion of Table 2: "The effect upon the wind-reaction ratio of a change in the relative size of girders is much more pronounced than is the effect of a similar change in columns."

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[16] Prof., Civ. Eng., Ohio State Univ., Columbus, Ohio.

[16a] Received by the Secretary September 11, 1941.

[3] "Wind Bracing," by Henry V. Spurr, M. Am. Soc. C. E., McGraw-Hill Book Co., Inc.

[17] *Proceedings*, Am. Soc. C. E., May, 1928, p. 1395; see also *Bulletin No. 48*, Eng. Experiment Station, Ohio State Univ., Columbus, Ohio.

It is also at variance with the facts, as shown by the following computations.[18] Fig. 3 shows the elastic lines of the members at a floor with the deflections greatly exaggerated. The drift due to column bending is ϕh and that

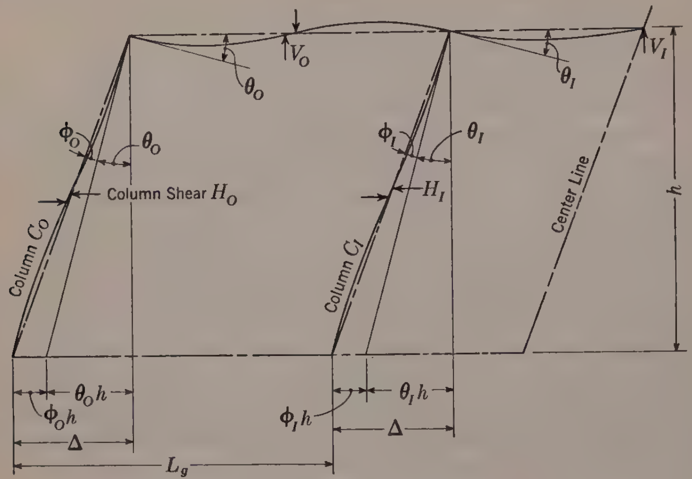

FIG. 3

due to girder bending is θh. From area moments,

$$\phi = \frac{H_c\, h^2}{12\, E\, I_c} \qquad \text{(4a)}$$

and

$$\theta = \frac{V_g\, L^2_g}{12\, E\, I_g} \qquad \text{(4b)}$$

Using the stresses and member sizes given by the author for the first floor: For column C_O(12-in. 85-lb H), $\phi_O = \frac{10.47\ (2 \times 8 \times 12)^2}{12\ E\ 723.3} = \frac{4.44}{E}$; for column C_I (14-in. 237-lb H), $\phi_I = \frac{39.53\ (2 \times 8 \times 12)^2}{12\ E\ 3{,}080.9} = \frac{3.94}{E}$; for girder G_O (24-in. 74-lb WF), $\theta_O = \frac{16.55\ (20 \times 12)^2}{12\ E\ 2{,}033.8} = \frac{39.0}{E}$; and for girder G_I (36-in. 160-lb WF), $\theta_I = \frac{34.6\ (30 \times 12)^2}{12\ E\ 9{,}738.8} = \frac{38.4}{E}$.

Thus, for the author's design, the effect of girder flexure on deflection is about ten times as great as the effect of column flexure. If the story height were about 12 ft instead of 20 ft, as is usual in most buildings, the effect of column flexure would be still less.

When it is desirable to reduce the deflection of the bent, the girder stiffnesses should be increased. The column flexure has little effect upon the total deflection. The deflection due to direct lengthening and shortening of the columns (chord deflection) is still smaller.

[18] *Bulletin No. 93*, Eng. Experiment Station, Ohio State Univ., Columbus, Ohio.

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DISCUSSIONS

A DIRECT METHOD OF FLOOD ROUTING

Discussion

By Messrs. Ray K. Linsley, Jr., Harold C. Hickman, Robert B. Horonjeff and Herbert G. Crowle, L. K. Sherman, and Alfred L. Brosio

Ray K. Linsley, Jr.,[5] Jun. Am. Soc. C. E.[5a]—In all engineering work the procedure adopted for any particular work is determined by the desired accuracy, available working data, purpose of the work, and other considerations. The routing method presented by the authors requires only a small amount of data and therefore may be extremely useful if it meets the other requirements of a particular problem. Some further consideration of the proposed method seems necessary before it can be evaluated properly.

The familiar storage equation, which serves as the basis for nearly all flood routing methods, is an exact formula. There are no simplifying assumptions in the statement "inflow minus outflow equals change in storage." It is not surprising, therefore, that the authors report close agreement between the computed volume of local inflow from the intervening area and the difference in volume of flow at the upstream and downstream stations. Working as they have from zero storage at the beginning of a flood to zero storage at the end of a flood, it would be impossible to have any error in volume other than an arithmetic one.

It is likewise to be expected that the computed hydrograph at the downstream station, obtained by combining the computed local inflow and the observed upstream flow, should check quite closely with the observed hydrograph at the lower station. The volume of flow, of course, should check exactly. Any difference in shape between the computed and observed hydrographs results from the procedure followed. In computing the local inflow, the reach storage was determined by using the sum of the observed inflow and outflow as an index of storage. In the recombination of the hydrographs, the inflow at the upper end and the local inflow are routed separately, using storage values dependent on the particular flow quantities. The process followed is that of

Note.—This paper by C. O. Wisler, M. Am. Soc. C. E., and E. F. Brater, Jun. Am. Soc. C. E., was published in June, 1941, *Proceedings*.

[5] Care U. S. Weather Bureau, Sacramento, Calif.

[5a] Received by the Secretary July 9, 1941.

dividing the total inflow into a reach into two parts and then adding these parts to get the whole. The error shown in Fig. 4 cannot be explained by a possible error in the Front Royal rating. The changes a flood wave undergoes are dependent on the total storage occupied within a reach. It does not seem rigorous, therefore, to divide the inflow arbitrarily into two parts and route these parts through separate and lesser storage volumes. Because the storage curves used are straight lines, this procedure can be used without introducing serious error.

A mathematically correct solution of the storage equation guarantees a true volumetric answer. The major problem in routing, therefore, is that of obtaining the correct distribution of the stream flow throughout the hydrograph. This requires accurate values of the storage changes during each routing period. True storage volumes are not necessary as long as the changes in storage can be determined correctly. Reviewing the proposed method in the light of known facts about stream flow, it is apparent that certain simplifying assumptions have been made which must of necessity affect the accuracy of the method to a greater or less degree, depending on the characteristics of the particular reach to which it is applied.

Assuming that there is no change in the channel characteristics of a reach during a flood, the storage within the reach is fixed by the water-surface profile. If this profile is a straight line, the stages at the two ends of the reach offer the most accurate measure of the storage within the reach. The authors have used the sum of the discharges at the end points of the reach as their index of storage volumes. This presumes a single-valued relation between stage and discharge, which is not strictly rigorous, although the error is not great for headwater stations such as were used in the examples. The actual water-surface profile in a reach is rarely a straight line during the passage of a flood wave. This fact probably introduces the greatest error in routing. In effect, the authors have used the mean discharge in the reach as their index of storage. Therefore, they are unable to distinguish between the rise and fall of the flood wave; yet it is well known that the shape of the water-surface profile, and therefore the storage volumes, may differ considerably between the rising and falling phases for a given mean discharge. The use of the channel recession to determine storage volumes permits defining the storage curve for falling stages only. The use of stages or discharges at the end points of a reach as separate values by means of a family of curves[6] more effectively defines the slope in a reach than does the mean stage or discharge. Weighting Q and O to give a weighted value of $Q + O$ for the storage index[7] provides a very practical way of adjusting for the effect of rise and fall.

The procedure proposed has not been put to a test effectively by the methods outlined in the paper. The routing of a flood through a reach with large amounts of ungaged local inflow permits only an estimate of the reasonableness of the computed local inflow hydrograph. Test routings of special cases offer an opportunity for a check on the accuracy of a routing method. One such case might be a release from a reservoir without concurrent rainfall. In this

[6] "Flood Routing," by Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 275.

[7] Discussion by Ralph W. Powell, *ibid.*, p. 298.

case the local inflow will be low and nearly constant, thus permitting a good comparison between observed and computed hydrographs at a downstream station. Similarly, if a river reach within which a large part of the tributary flow is gaged can be found, the local inflow hydrograph can be computed from the upstream and downstream stations only, and this computed hydrograph can then be compared with the total gaged tributary flow adjusted for the ungaged inflow.

The authors suggest that, by prorating the storage volumes and the local inflow on a basis of drainage area, hydrographs for intermediate points of a reach can be computed by routing. Channel storage is a function of channel characteristics and need not show any relation to drainage area. Local inflow volume is controlled by rainfall distrubution and the runoff characteristics of the area under consideration. The shape of the local inflow hydrograph is a function of rainfall distribution with respect to both time and area and to the shape of the contributing area. It would seem advisable to avoid any arbitrary adjustment of storage and flow purely on the basis of drainage area. Contour maps and channel cross sections can be of considerable assistance in the prorating of storage, and the synthetic unit graph[8] can be of much help with the tributary flow.

Harold C. Hickman,[9] Assoc. M. Am. Soc. C. E.[9a]—Flood routing based on accurate and substantial information is at best an intricate and laborious problem. The method described in the paper is particularly interesting in that no cross sections of stream channels, no velocities of flow, and no discharge records on the tributaries are needed. The entire procedure is based upon the storage equation and upon the principle that, for all high stages, there is a straight-line relationship between the volume of storage contained in any reach of the river channel and the sum of the inflow rate at the upper end and the outflow rate at the lower end of that reach. To continue further, it is assumed that at any point downstream the total flow at any instant is comprised of water contributed by many different tributaries, and that the percentage of the total flow occurring at any instant, that is contributed by any single tributary, is constantly changing throughout the flood period.

In analyzing the procedure for the determination of unmeasured inflow, it has been assumed that for any given reach of river channel there is a definite relationship between the volume of channel storage contained therein at any instant and the sum of the simultaneous discharge rates at the two ends of the reach. There is no doubt that this relationship exists, because the discharge is proportional to the river stage at the upper end of the reach, the outflow is proportional to the stage at the lower end, and discharge plus outflow is proportional to the mean stage which, in turn, is dependent on the volume stored in the reach. The relationship between these two quantities varies as a straight line, but, in obtaining the storage in the reach by measuring the area beneath the hydrograph, only a small section of the hydrograph can be used.

[8] "Synthetic Unit Graphs," by F. F. Snyder, *Transactions*, Am. Geophysical Union, 1938, p. 447.

[9] Asst. Engr., U. S. Engr. Office, War Dept., Detroit, Mich.

[9a] Received by the Secretary July 23, 1941.

In other words, this relationship holds good only after the stream has obtained "equilibrium" and is not influenced by overland flow. The entire procedure depends upon the relationship between $(Q + O)$ and the storage in the intervening reach. There is some doubt as to whether a solution should be based on a relationship of this kind.

Assuming for the present that the theory is rational, the next step is to route Q and I through the system to Morgantown. For this purpose, the inflow from the intervening area, I, was ignored, and the storage formula, Eq. 1, was written as in Eq. 3. The quantities on the left-hand side of Eq. 3 are known, giving a quantity which is the summation of $S_1 + O_1$. If Q_1 is added to each side, Eq. 4 results, which was solved by means of the curve in Fig. 2.

The relationship was determined by introducing I into Eq. 1 and therefore cannot be used as a means of solving Eq. 4. In other words, the manner in which the flood peak moves downstream, if the intervening area were not there, is decidedly different from the manner in which the peak will move downstream under the influence of the discharge of the intervening area. On the other hand, if the routing is accomplished by moving the flood peak, as influenced by the discharge of the intervening area, downstream, the problem resolves itself into the simple procedure of segregating the flood peak at Morgantown into its two component parts, depending on the stage-discharge relationship at Fetterman.

In comparing the foregoing method of flood routing with other methods, certain facts must be taken into consideration. In the former method, only stream-flow records during a typical flood at a few points on the main stream, or on the tributaries whose flow is to be routed downstream, are necessary. It is universally accepted that when a known volume of water falls on the surface of a drainage area, the rate of resulting river flow at the lower end of the drainage area (point P) is influenced by many complex factors. Some of these, such as the consistency and moisture control of the soil, determine how much water will become surface runoff and enter the stream in time to contribute to the flood. Topography, valley storage, and channel storage are all factors in determining the shape of the hydrograph at P. After surface runoff has ceased, valley storage governs the flow. When dealing with small homogeneous areas, the evaluation of these factors is needless because the unit hydrograph method evaluates their combined effect as a single entity. With larger and more complex areas there is some question regarding accuracy.

Flood routing divides itself into two distinct problems and combinations of the two: (1) Flood synthesis, which is the determination of the magnitude and time of concentration of flood peaks on the various tributaries; and (2) flood routing, which is the determination of the manner in which these peaks travel and combine themselves with other peaks from other tributaries. Flood routing can be defined further as the procedure by which the hydrograph at any point on a stream is determined from a known hydrograph at some point upstream. As explained, the term "routing" is commonly used to include not only the modification of the flood wave caused by progress downstream, but the further modification caused by any additional flow that enters the river from tributaries or local runoff during this progress.

Factors Affecting Travel.—If a flood hydrograph is divided into elements of equal time, it can be said that certain factors affect the travel of the various elements and alter the shape of the flood wave as it travels downstream.

At the beginning of the flood hydrograph, the first few elements represent a part of the uniform flow of the stream, and they tend to maintain a condition of uniform flow. The elements following, which pass during the rise, have greater slopes and hydraulic radii than the normal river, and, since $V = C\sqrt{R\,S}$, their velocities are greater than the normal. As the stages rise, part of the discharge is temporarily lost in valley storage. As the flow at the lower end of the reach decreases, the valley storage in the reach will flow out and add to the discharge.

Valley storage in a reach at any time is equal to the inflow minus the outflow, and maximum storage occurs at the point of intersection of the two flow hydrographs. The point of intersection also represents time elapsed, and prior to this time the inflow exceeds the outflow and storage increases; after this, the outflow exceeds the inflow and storage is reduced. The curves intersect at the maximum point on the outflow hydrograph and the areas between the two curves are each equal to the increase of valley storage in the reach. Several factors affect the shape of the flood wave. They may be listed as: Rate of rise, height of rise, slope of channel, stages downstream, downstream channel sections, length of reach, length of crest of hydrograph, and rate of fall.

(1) Rate of Rise.—A rapid rate of rise causes high velocities in the first stages of the flood, which in turn result in the rapid dissipation of these first elements in valley storage, and the consequent retardation of the peak of the flood wave.

(2) Height of Rise.—The higher the stage, the greater the valley storage will be, which in turn causes a flattening of the peak. This causes a greater coefficient of roughness, which reduces velocities.

(3) Slope of Channel.—Greater slopes of the channel will allow more rapid runoff, causing lower stages and less valley storage.

(4) Stages Downstream.—With stages low downstream, a large part of the flood wave goes into valley storage. Increases in downstream stages may cause spreading to the alluvial plain, and thereafter the loss in valley storage will again become large. The propagation and intensity of the peak will be lessened. As the downstream stages are falling at a rate equal to the rate of increase in the rising flood wave, the latter will pass without loss or change due to valley storage, and the stream will tend to maintain a uniform discharge.

(5) Downstream Channel Sections.—If channel sections increase proportionately downstream, loss of valley storage will be greater than if they remain uniform, and the height and rate of propagation of the peak will be correspondingly reduced. If channel sections decrease, the reverse is true.

(6) Length of Reach.—The longer the reach the greater will be the valley storage and the more the peak will be reduced and retarded. These effects vary as a fractional power of the length of reach. Mathematically, in a uniform channel of indefinite length, the rise would be propagated downstream indefinitely, and, since the rate of peak reduction is a finite quantity at the beginning

and can never become zero, the curve of successive peaks is concave upward and not a straight line.

(7) Length of Crest of Hydrograph.—The longer the crest, the less is the peak height affected by valley storage.

(8) Rate of Fall.—The more rapid the rate of fall, the more rapidly valley storage will flow out.

The effects that factors (1) to (8) have on a flood wave as it proceeds downstream are individually obvious. In order to route a flood, it is necessary to evaluate the combined effect of all factors. This presents a problem so complex that it defies exact solution by any known method.

A number of empirical methods have been used satisfactorily, but in all of them the stream has been divided into reaches. Since it is desirable to know the stage-discharge relationship at the downstream limit of each reach, the division is generally influenced by the location of gaging stations. Since the shape of the flood wave is constantly being warped by the flow from tributary streams, the accuracy of the computations is inversely proportionate to the length of the reaches. Reaches should be of such length that the simultaneous rates of rise or fall at their extremities are approximately equal and are of the same sense (the latter condition being impossible while the peak is passing through the reach). In the Allegheny and Monongahela rivers, reaches of from 10.6 to 80.7 miles have been used successfully.

All methods of flood routing are based on the solution of the basic equation:

$$I = O + \Delta S \qquad (6)$$

in which, for a given period of time T: I = total inflow; O = total outflow; and ΔS = change in storage. The time T should be no longer than the time of travel of the flood wave through the reach. Various methods, taking into consideration the aforementioned complex factors, have been developed and are generally accepted as giving good results.

Method of Inflow—Storage-Discharge Curves.—In general, the method consists of dividing the stream into reaches and determining the natural valley storage in each reach corresponding to the various discharges at the lower end of the reach. Assuming a straight line function for change of inflow and outflow during a time interval t, Eq. 6 may be written—

$$t\left(\frac{i_1 + i_2}{2}\right) = t\left(\frac{d_1 + d_2}{2}\right) + \Delta S \qquad (7)$$

in which: i_1 = inflow, in cubic feet per second, at initial instant of period; i_2 = inflow in cubic feet per second, at final instant of period; d_1 = discharge, in cubic feet per second, at initial instant of period; d_2 = discharge, in cubic feet per second, at final instant of period; ΔS = increase in storage, in cubic feet per second, during period; and t = length of period, in seconds.

In applying the method, consideration must be given to the flow that enters the reach from tributaries while the flood wave is passing along the reach. For a given value of discharge there will be several different values of valley storage, depending on whether the inflow enters all at one point, or at two points, or at a

greater number. If it enters from several different locations, there will be different values, depending on the relative proportions entering at these different locations. Strictly, there are an infinite number of such variations. For practical purposes, however, on any specific reach there may be distinguished a limited number of "types" of floods, depending on which tributary or combination is contributing most of the flow. Separate computations must be made for each different type.

The decision as to how many such general types of floods will be distinguished, of course, must be made by a skilled engineer from a study of the specific reach in question. Among other things, it will depend on the location of the reach. In headwater areas a large percentage of the inflow usually enters from tributaries, whereas lower down the river the flow in the main stream is usually the principal component of the total inflow into the reach.

The storage and discharge curves are obtained by plotting storage and discharge, respectively, against gage height. Daily inflow at the head of the reach and rated tributaries entering the reach are tabulated. From plots of the storm, the rainfall over the area draining directly into the reach can be determined. The local inflow is computed from these data by the distribution-graph method. The daily discharge, gaged at the lower end of the reach, is recorded. The total outflow from any particular flood must equal the total inflow. Ordinarily, they do not check exactly, since there is a greater chance for error in computing the inflow.

With these data, the accumulated storage in the reach can be computed. A time interval of one day is used, and it is assumed that 1 cu ft per sec for one day equals 2 acre-ft (a very close approximation). Eq. 7 can then be written—

$$(i_1 + i_2) - (d_1 + d_2) = \Delta S \qquad (8)$$

in which: i_1 and i_2 are numerically equal to the daily values of inflow in cubic feet per second, but are acre-feet in this equation; d_1 and d_2 are numerically equal to the daily values of outflow in cubic feet per second, but are acre-feet in this equation, and ΔS = increase in storage for the first day, in acre-feet. The curves are then plotted.

This method is used by the Mississippi River Commission to predict flood stages and the time of their occurrence at various points in the main river, resulting from known or predicted conditions of flow in the Mississippi and Ohio rivers at Thebes and Metropolis, Ill., and in the major tributaries downstream.

It is extremely difficult to evaluate all the varied factors that influence flow in a river such as the Mississippi. The computation of the data, although simple in principle, is very difficult in fact. In certain instances, assumptions regarding computations, such as those for evaluating the effect of local and tributary inflow, must be made.

Every such prediction, of course, is based on the assumption that no rains of major proportions will fall after the date on which the prediction is made. If such rains occur, new predictions must be made. For this reason, predictions should be published with caution, since the layman does not always understand their inherent limitations.

Steinberg Method.—A method was developed by I. H. Steinberg, junior engineer, U. S. Engineer Office, St. Paul, Minn., in connection with the 9-ft channel project on the upper Mississippi River.[10]

A group of storage-discharge curves are used for a certain reach of a certain river. They have been obtained by observations of past floods on the reach, and are the same as the inflow storage-discharge curve except that in this case storage is plotted against discharge. There are five different curves, corresponding to five different conditions of backwater resulting from the inflow of a large tributary just below the reach, the volume of that tributary's flow being the parameter of each curve. (This is analogous to the development of several sets of curves in the inflow storage-discharge method, one for each "type" of flood.)

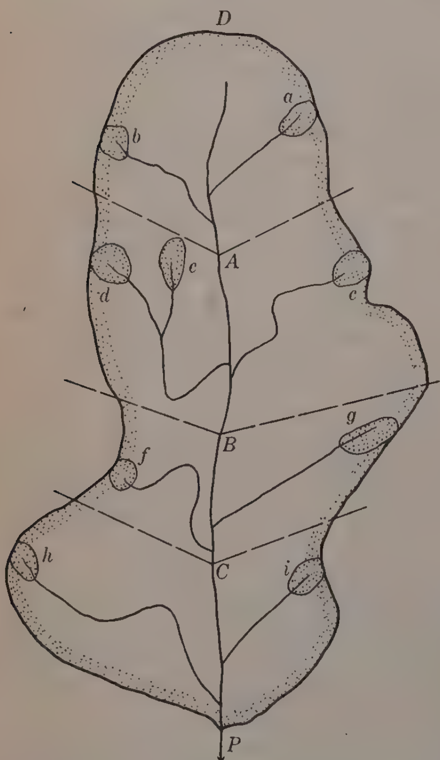

FIG. 5

Starting with the fundamental Eq. 7, writing $\Delta S = S_2 - S_1$, and transposing:

$$\frac{t}{2}(i_1 + i_2 - d_1) + S_1 = \frac{t}{2} d_2 + S_2 . \quad (9)$$

All the knowns are on the left-hand side of Eq. 9, and the two unknowns, d_2 and S_2, are on the right-hand side. Call the left-hand member of the equation the "storage factor," and represent it by M; and let $t = 1$. Then: $M = \frac{d_2}{2} + S_2$ which, transposed, equals:

$$S_2 = M - \frac{d_2}{2} \quad \ldots\ldots (10)$$

If Eq. 10 is plotted, S_2 against d_2, it is a straight line with a slope of -2 and an intercept of the S-axis of M. (Had t been given some other value than 1, the slope, of course, would have been different.) For different values of "M," a family of parallel straight lines may be plotted. Given the foregoing data, and certain basic data, flood routing may be accomplished by simple computations and by interpolation on the set of curves.

Other Methods.—Other methods of flood routing have been used successfully and the Muskingum method is one of them. It is based on the assumption that the ratio of valley storage within a reach to a weighted flow factor determined from both inflow and outflow is constant, and is dependent upon the physical

[10] Civil Engineering, July, 1938, p. 476.

shape of the valley within the reach. This is not theoretically correct, but is a close approximation within a limited range of flood stages.

There is another method that is a combination of the unit hydrograph with flood routing. Where the drainage area is too large to be dealt with by a single unit hydrograph, flood stages are forecast directly from rainfall data. For rivers such as the Connecticut, where the flood wave travels downstream very rapidly, predictions based on rainfall are usually the only ones that can be made far enough in advance to be of much use.

Assume the area *PD*, Fig. 5, to be drained by the system of streams shown. The method of forecasting stages at *P* is as follows:

At points *A*, *B*, *C*, and *P*, unit hydrographs were constructed. These are the hydrographs resulting from 1 in. of runoff over the successive divisions of the total drainage area—that is, the hydrograph at *A* is for the runoff from area *AD*, that at *B* is for the runoff from area *BA* alone, that at *C* is for the runoff from area *CB* alone, etc. The river was considered to be in flood, and these hydrographs were routed separately to *P*. Curves *A′*, *B′*, *C′*, and *P′*, Fig. 6,

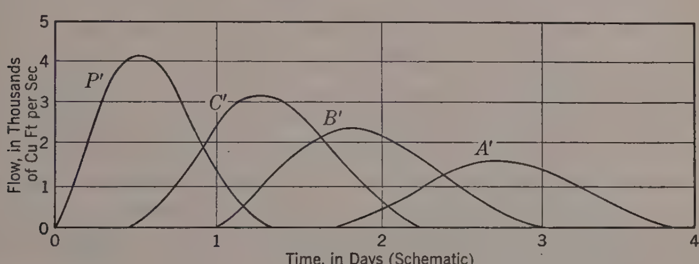

Fig. 6

are the resulting hydrographs at *P*. They show the magnitudes of the elementary flows at *P* resulting from the unit hydrographs at the other points, and also the time lag between the appearance flow at point *A*, *B*, or *C* and of the resultant flow at *P*. The rain is assumed to have started at zero time.

Gaging stations are installed at points *a*, *b*, *c*, *d*, etc. The small stippled areas are the index areas for the subdivisions. The runoff factors for the index areas are applied to the rainfall on the larger areas in order to determine the number of inches of runoff from these areas. The ordinates of the curves *A′*, *B′*, *C′*, and *P′*, which show the resulting flow at *P* from a runoff of 1 in. over areas *AD*, *BA*, *CB*, and *PC*, respectively, are multiplied by the runoffs in inches as thus computed. The curves are then compounded to obtain the hydrograph at *P* that will result from the given storm.

In routing the individual unit hydrographs at *P*, the river was assumed to be in flood. The operation is therefore accurate only when this condition prevails. It would be erroneous to assume, for example, that the flow at *P* resulting from a 2-in. runoff from area *AD* alone could be found by multiplying the ordinates of curve *A′* by 2. Since no rain fell on areas *PC*, *CB*, and *BA*, the stages of the stream in those areas would be normal when the flood waters

from upstream arrived, and much of the flood flow would be taken up in valley storage. The peak at P would therefore occur later than curve A' indicated, and would be lower than the value obtained by multiplying the maximum ordinate of A' by 2.

Conclusions.—It can be noted from the foregoing that flood routing with all of its complex factors cannot be considered in the light of the information from one or two flood hydrographs. When seven or eight factors, each having considerable individual effect on the size and shape of the flood hydrograph, join together in different combinations, depending upon the assimilation of predetermined conditions, the resulting effect on the flood hydrograph may be unpredictable unless all of the information that can possibly be collected is analyzed and given consideration in a manner suitable to its importance to the problem.

It is believed that the method described by Messrs. Wisler and Brater is unique and should be tested under varying conditions. It may be that this method eliminates most of the tedious work connected with the routing of floods in a large and intricate drainage system. However, the most serious objection to the method is that it is based on the assumption that routing within a basin may depend on what has happened during one flood. Can it be said that a major storm, and the resulting flood, will proceed according to the same plan every time? Unless it is true that all of the factors affect all floods on a particular stream in the same manner and to the same degree during every storm, there is no reason why each flood will not be different from its predecessor even though the storm producing it begins under identical conditions. In other words, every storm should be studied separately, so that each factor affecting the flood can be isolated and compared if possible.

Any method, in which the contributing factors can be used in different combinations to determine the resulting effect, appears to be more rational than a method that proposes to assume that all floods proceed according to the same plan under all combinations of meteorological, channel, and basin conditions.

ROBERT B. HORONJEFF,[11] AND HERBERT G. CROWLE,[12] JUNIORS, AM. SOC. C. E.[12a]—The flood-routing technique presented in this paper may prove to be of considerable value, provided the full significance of the various operations is understood by those who use the method.

The writers are in complete agreement with the basic equations presented, but they believe that the authors have not given sufficient explanation of the following features:

(1) The expression "storage in the intervening reach" includes the storage in all channels tributary to the reach as well as the storage in the main channel. This fact must be kept in mind in any attempt to visualize the proportionality of "storage in the intervening reach" and the "sum of the inflow rate at the upper end and the outflow rate at the lower end of that reach."

[11] Associate Engr., U. S. Engr. Office, San Francisco, Calif.

[12] Asst. Engr., U. S. Engr. Office, San Francisco, Calif.

[12a] Received by the Secretary July 29, 1941.

(2) Referring to Fig. 1, the agreement of the composite of the routed hydrographs with the gaged hydrograph O at the lower end of the reach is merely a check on the arithmetical operations performed in deriving the hydrograph marked I, and in routing I and Q through the reach with the straight-line relation between S and $(Q + O)$ shown in Fig. 2. The agreement is in no sense a check on the accuracy of the gaging at the lower station, or the correctness of the lines in Fig. 2.

As proof of the foregoing statement, the writers present the set of hydrographs for the Pajaro River, California, in Fig. 7, corresponding to Fig. 1, and

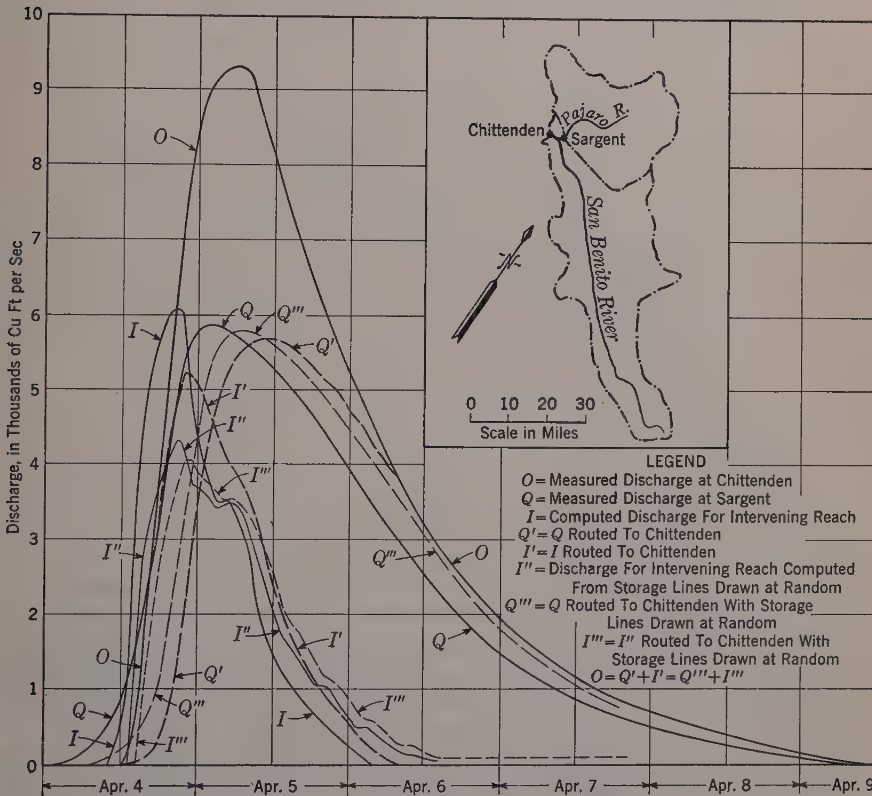

FIG. 7.—HYDROGRAPHS OF SURFACE RUNOFF, WATERSHED OF THE PAJARO RIVER, CALIFORNIA

the inflow-outflow-storage lines in Fig. 8, corresponding to Fig. 2. A characteristic flood is routed from Sargent to Chittenden, Calif., discharge records being available at both of these places. The hydrographs marked I, I', and Q' were obtained by the methods outlined in the paper, using the "S versus $(Q + O)$" and "$S + (Q + O)$ versus $(Q + O)$" curves derived from the recession curves of hydrographs Q and O. The sum of the hydrographs I' and Q' was so close to the hydrograph O as to be indistinguishable. In Fig. 8, a second "S versus $(Q + O)$" curve was then drawn at random, with the corresponding "$S + (Q + O)$ versus $(Q + O)$" curve, and the hydrographs I'', I''', and Q''' derived with these new lines. The curve I'' represents

the fictitious hydrograph of inflow from the intervening reach, I''' that hydrograph routed through the reach, and Q''' the fictitious routed hydrograph of Q. The sum of I''' and Q''' agrees exactly with the original gaged hydrograph O, proving that the agreement depends only upon the use of a straight line—any straight line—for the routing operation. This is perhaps the most important criticism of the paper—namely, that the agreement of the composite of the routed hydrographs with the gaged hydrograph does not prove the accuracy of the results in a physical sense.

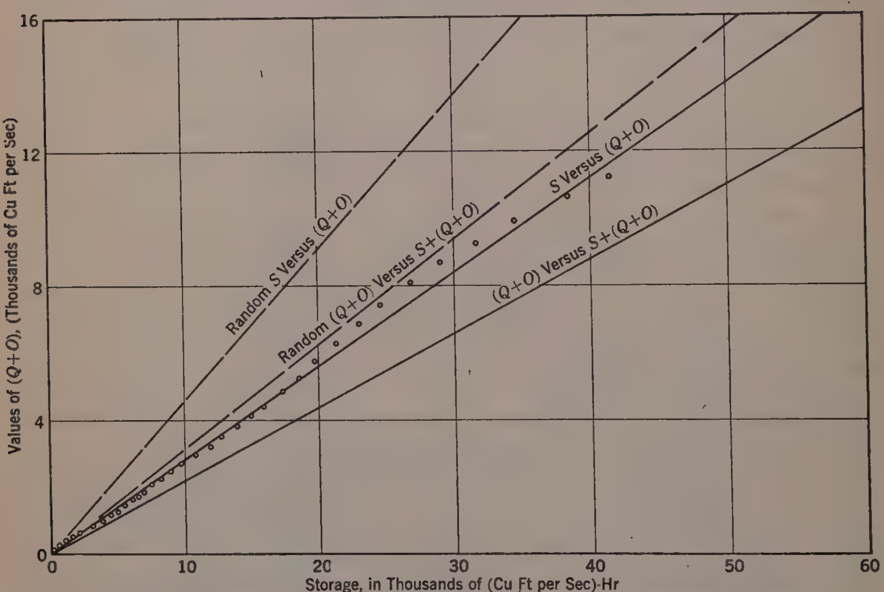

Fig. 8.—Relation Between $(Q + O)$ and the Storage in the Intervening Reach, Pajaro River, California

(3) The authors did not indicate whether the storage curve derived from one flood would plot in very nearly the same position as those obtained from other floods. The writers found, in the case of the Pajaro River, that "S versus $(Q + O)$" curves, which were derived from different floods, plotted in different positions and with appreciably different slopes, indicating that there is no one curve that is characteristic of a particular reach during all floods.

(4) Base flow may be included in the routing operation, thus eliminating the necessity of adding this value to obtain the complete routed hydrograph at the lower end of the reach.

(5) Finally, the writers wish to emphasize that the authors so far have presented no evidence to prove that their method of routing gives the correct reduction in peak discharge to be expected from a reservoir constructed on a tributary. In fact, it appears to the writers that the nature of the method precludes the possibility of a check upon the physical results. Any errors in gaging or in deriving the "S versus $(Q + O)$" curve are automatically included in the derivation of the hydrograph of computed discharge I for the intervening reach.

L. K. SHERMAN,[13] M. AM. SOC. C. E.[13a]—The writer has tested the authors' flood routing method by utilizing an example published in 1940.[14] The gaged stations, reported by the authors, at Morgantown and Fetterman correspond with the two points at the ends of the lower (6-ft base) reach of the main channel, shown in Fig. 9.[15] The authors' hydrograph O corresponds with "(b) Heavy Rain," in Fig. 10. The hydrographs, corresponding to the authors' Q and routed Q_1, are plotted as curves d and e in Fig. 10.

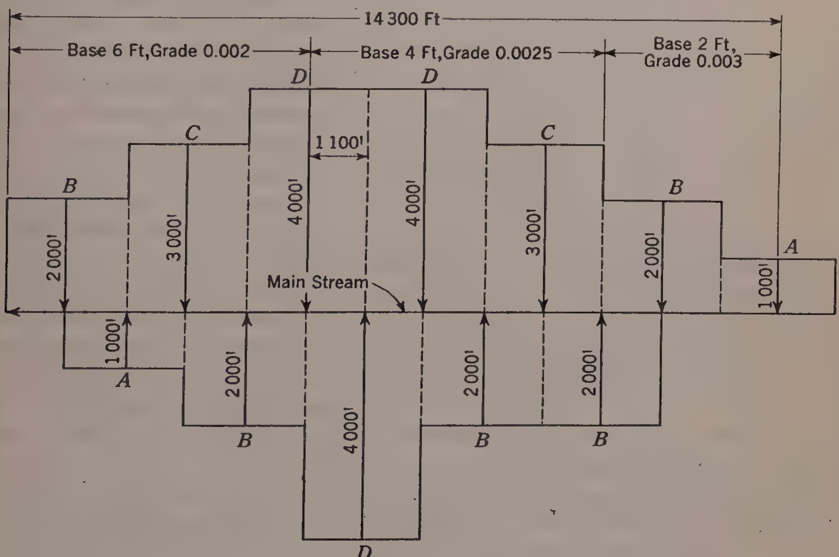

FIG. 9.—THE DRAINAGE AREA

The relation of cubic feet per second at the outlet station, to the valley storage, was determined from recession curves of the hydrographs O, Q, and also from the small hydrograph in Fig. 10 marked "(a) Light Rain." The points were platted on logarithmic paper similar to the diagram given by Robert Horton,[3] M. Am. Soc. C. E. All of the points below the contraflexure on the recession curves of the hydrographs fell on a straight line, except low values of cubic feet per second. In this example, all surface runoff ceased at the time 8,400 sec, as shown by the hydrograph of overland flow,[16] Fig. 11, for one unit of area. This time checks with the points of contraflexure. The total overland flow from the "heavy rain" on one unit strip of 1,100 ft is 71 cu ft. The total volumes of runoff, for checking purposes, are: Hydrograph Q or Q_1 is 44,000 × 71 cu ft. The volume I for the reach is 16,000 × 71, and the volume of hydrograph O is 60,000 × 71.

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[13] Cons. Engr., Chicago, Ill.

[13a] Received by the Secretary September 2, 1941.

[14] "The Hydraulics of Surface Runoff," by L. K. Sherman, *Civil Engineering*, March, 1940, p. 165.

[15] *Ibid.*, Fig. 1.

[3] "Surface Runoff Phenomena," by Robert E. Horton, Publication 101, Horton Hydrological Lab., Voorheesville, N. Y., 1935, p. 35.

[16] "The Hydraulics of Surface Runoff," by L. K. Sherman, *Civil Engineering*, March, 1940, p. 165, Fig. 3.

The writer platted a diagram of $(Q + O)$ versus storage, with time units of 250 sec, corresponding with Fig. 2. The fifteen points all fell on, or close to, a straight line through the two points $(Q + O) = 150$ cu ft per sec versus $S = 392$ cu ft per sec (250 sec) and $(Q + O) = 1{,}353$ cu ft per sec versus $S = 3{,}260$ cu ft per sec (250 sec). The line $(Q + O)$ versus $S + (Q + O)$ was also platted.

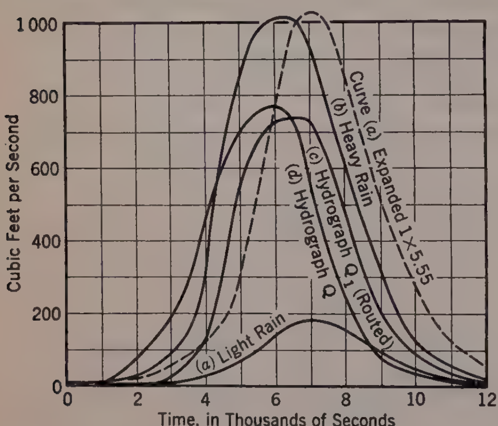

FIG. 10.—MAIN CHANNEL HYDROGRAPHS FOR BOTH LIGHT RAIN AND HEAVY RAIN

The writer's routed hydrograph Q_1 was derived with this diagram and with Eq. 4. The tails of his recession curves Q_1 and O did not coincide as do the similar curves of the authors. The writer knows of no reason why they should coincide. Even with equal durations of rainfall, the recession curve of the hydrograph with the lesser peak and volume will fall inside the larger. As an extreme case, note the two hydrographs in Fig. 10.

Conclusions.—(1) The authors have presented a reliable method that will be useful in many problems of flood routing.

(2) In selecting recession curves for the relation between discharge rate and storage, avoid, if possible, hydrographs affected by rainfall or surface runoff during the recession period.

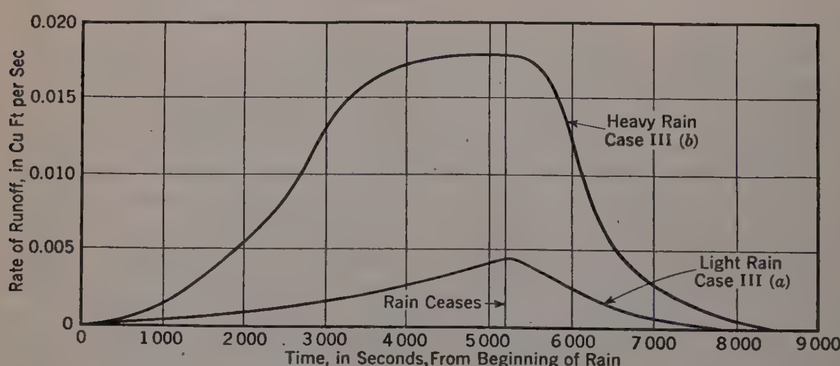

FIG. 11.—OVERLAND FLOW FROM A 1-FT STRIP 1,100 FT LONG, WITH DETENTION STORAGE

(3) The straight-line relation for $(Q + O)$ versus storage exists for a longer period than the similar line for discharge and storage.

A supported hydrograph (that is, one receiving increments of water as it moves downstream) may be routed by Seddon's wave velocity if the channel areas are known. The routed hydrograph will check with the storage equation

at any time. A hydrograph routed by stream velocity will not check with the storage equation. The total volumes will agree, but the forms of the hydrographs will not agree.

An attempt was made by the writer[14] to compare the known volume of the reach with the volume derived by the difference of total hydrograph volumes, but it was not too successful.

It would appear that, on natural streams, more reliable values of storage could be derived by the authors' method than by cross-section surveys.

Alfred L. Brosio,[17] Jun. Am. Soc. C. E.[17a]—The method of flood routing that the authors have presented involves assumptions that need further explanation. The comments which this writer wishes to present will be separated into two parts—I. Discussion of the assumption of a straight-line flow-storage relation; and II. Analysis of the assumption: Storage = function (inflow + outflow).

I. Assumption of Straight-Line Flow-Storage Relation.—In the "Conclusion," the authors have stated their belief that "except possibly under unusual channel conditions, the curve of relationship between channel storage in the intervening reach and the sum of the discharges at the two ends of the reach is a straight line for the normal and high stages of the river." Although such a relationship is sometimes found to exist in natural channels, it is not believed to represent the "usual" channel conditions. This can be shown as follows: Let S = volume of channel storage in second-foot-hours; Q_u = uniform rate of flow, through entire reach, in cubic feet per second. A straight-line relationship between S and (inflow + outflow) implies that $\frac{S}{Q_u}$ = constant; but a constant value of $\frac{S}{Q_u}$, which is in units of time, represents a constant time of travel, and, consequently, a constant mean velocity of flow irrespective of stage. Therefore, the straight-line relationship should not be assumed to apply in the case of channels having such cross section that mean velocity is a function of depth of flow. Such channels are not believed to be "unusual."

The authors report that "in every application made to date (1941), this straight-line relationship holds true * * * for values derived to the right of the point of contraflexure." The writer suggests that perhaps this finding is a direct result of the assumptions involved. An analysis of the authors' procedure in the Monongahela River example will clarify the implications of the last sentence. Let: S_m = total channel storage above Morgantown; S_f = channel storage above Fetterman; S_a = total storage in the channels of the "intervening area"; S = storage in the Fetterman-to-Morgantown channel; S_i = storage in the channels of the "intervening area," exclusive of S (the intervening area comprises 50% of the total area above Morgantown, and many miles of channel are included therein in addition to the Fetterman-to-Morgantown channel); O = Morgantown flow; Q = Fetterman flow; and I = flow from the "intervening area."

[17] Asst. Engr., U. S. Engr. Office, Sacramento, Calif.

[17a] Received by the Secretary September 15, 1941.

The example is somewhat complicated because of rain that fell from March 26 to 28, during the period of recession which the authors chose for their study. However, the writer does not contest the statement of the authors to the effect that, had not this additional rain fallen, values of $(S_m - S_f)$ computed from the respective hydrographs subsequent to the point of contraflexure would have plotted against $(Q + O)$ as a straight line; but $S_m - S_f = S_a$; and $S_a = S + S_i$. Therefore, by adopting a curve conforming with plotted points of $(S_m - S_f)$ versus $(Q + O)$, to represent S versus $(Q + O)$, the authors in effect assumed S_i, and consequently I, to equal zero. Going on to the next step in the authors' procedure, the hydrograph of I was developed, using the adopted flow-storage relation, and found to equal zero during the periods in which it had been so assumed by implication; but it is obvious that had any other assumption been implied with regard to the value of I, the results would have confirmed such assumption quite as well. For example, the writer has made a test in which a curve of S versus $(Q + O)$ was adopted conforming with the authors' curve up to $(Q + O) = 10{,}000$ cu ft per sec and having a "concave-upward" shape thereafter. The assumption implied by such a curve is that S_i did not deplete itself until $(Q + O)$ receded to a value of 10,000 cu ft per sec, shortly before noon on March 29. The hydrograph of I, developed as required by the authors' equations and using the concave curve, confirms the implied assumption of the writer, although it is understood that such assumption is probably no better than the one implied by the authors. The detailed calculations and results of this test are not presented herewith, but a similar test may be made in a short time by any one desiring to verify the foregoing statements.

It appears, therefore, that the method used by the authors to compute channel storage may be somewhat unreliable. Unfortunately, the writer is not at all familiar with the streams studied by the authors, nor with the quantity and quality of meteorological and hydrological data available. However, assuming the existence of a very minimum of such data, the writer would recommend the following general procedure to obtain the volume of storage in a channel such as the one from Fetterman to Morgantown:

1. Choose for analysis that flood, for which both Fetterman and Morgantown flows are known, which offers the best combination of the following: (*a*) Precipitation data in, or near, the "intervening area"; and (*b*) a minimum percentage of total runoff above Morgantown originating from the "intervening area."

2. Using the method devised by Otto H. Meyer,[18] Assoc. M. Am. Soc. C. E., for determining hydrographs for areas lacking in runoff records, the hydrograph of flow from the intervening area during the flood chosen for analysis may be estimated as to shape, total volume being known from the difference "Morgantown minus Fetterman." Should the basic data used in conjunction with Mr. Meyer's method be very poor, the choice of a flood having the quality of Item 1(*b*) will be of definite advantage.

[18] "Analysis of Run-Off Characteristics," by Otto H. Meyer, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), pp. 83–100.

3. Storage in the channel Fetterman-to-Morgantown may then be computed at any instant, knowing total inflow, as well as outflow, during any interval of time. This channel storage may then be used in further studies, to determine its relation to other variables. Since Mr. Meyer's method makes available not only the hydrograph I subsequent to the point of contraflexure, but the entire flood hydrograph, larger values of channel storage may be computed than from a method limited to analysis of recessions.

Of course, it is not to be understood that the method outlined herein to compute channel storage is the only acceptable one. With more basic data the selection of methods becomes correspondingly greater. It is possible that had the authors determined channel storage in the various reaches by a more reliable method, the correlations of S versus (inflow + outflow) might not have yielded straight lines.

II. Storage as a Function of (Inflow + Outflow).—The statement (see heading "Determination of Unmeasured Inflow") that "there is a definite relationship between the volume of channel storage * * * at any instant and the sum of the simultaneous discharge rates at the two ends of the reach" sounds very logical at first thought. However, it is an assumption, the weaknesses of which should be fully understood.

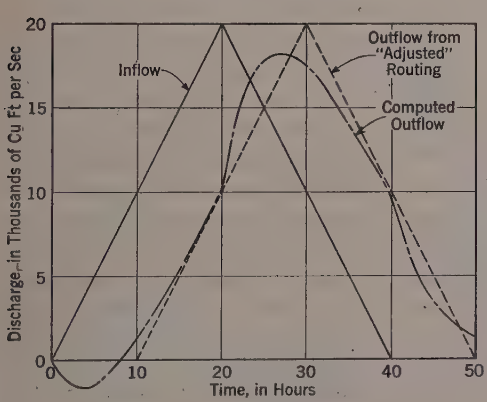

FIG. 12

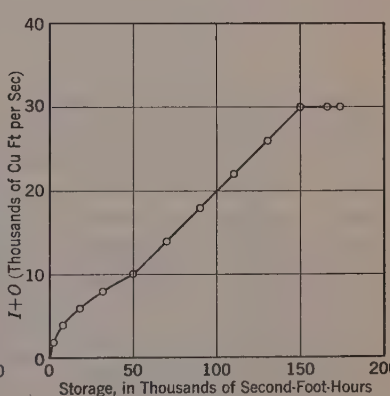

FIG. 13

Let the hydrograph represented by the full line of Fig. 12 be the total flow into a reach of channel, entering entirely at the upper end of the reach. Also, assume that the relationship between storage and uniform flow is a straight line, having a value of inverse slope equal to 10 $\frac{\text{second-foot-hours}}{\text{second-foot}}$, or 10 hr. If the inflow is routed through the reach according to the method presented by the authors, using 2-hr periods, the outflow hydrograph indicated in Fig. 12 by the dash-dot symbol is the result. There are three apparent inconsistencies in that hydrograph: (1) Outflow is negative during the first 8 hr; (2) rate of

rise of outflow suddenly increases at Hour 20, when inflow begins to fall; and (3) rate of fall of outflow suddenly increases at Hour 40, when it would be expected to do the opposite.

The writer does not believe that a detailed explanation is required, relating to the foregoing inconsistencies. It is obvious that the assumption storage = function (inflow + outflow) assumes a uniformly varying flow between the two ends of the reach and that such a condition does not exist during the three periods of discrepancy. The same types of inconsistencies occurred in the examples presented by the authors. Some good illustrations may be found in the routing of Q into Q', in Fig. 1. The writer believes that the authors must also have obtained negative values of flow, not shown in the hydrographs of Figs. 1 and 4.

TABLE 2.—ADJUSTED ROUTING COMPUTATIONS FOR THE TRIANGULAR-SHAPED INFLOW HYDROGRAPH OF FIG. 12

Hour	I	O	S	Eq. 11	$I_1 + O_1$
0	0.0	0.0	0.0		
2	2.0	~~−1.3~~ 0.0	2.0	4.0	0.7
4	4.0	~~−2.0~~ 0.0	8.0	12.0	2.0
6	6.0	~~−2.0~~ 0.0	18.0	24.0	4.0
8	8.0	~~−1.3~~ 0.0	32.0	40.0	6.7
10	10.0	0.0	50.0	60.0	10.0
12	12.0	2.0	70.0	84.0	14.0
14	14.0	4.0	90.0	108.0	18.0
16	16.0	6.0	110.0	132.0	22.0
18	18.0	8.0	130.0	156.0	26.0
20	20.0	10.0	150.0	180.0	30.0
22	18.0	~~14.7~~ 12.0	166.0	196.0	32.7
24	16.0	~~18.0~~ 14.0	174.0	204.0	34.0
26	14.0	~~20.0~~ 16.0	174.0	204.0	34.0
28	12.0	~~20.7~~ 18.0	166.0	196.0	32.7
30	10.0	20.0	150.0	180.0	30.0
32	8.0	18.0	130.0	156.0	26.0
34	6.0	16.0	110.0	132.0	22.0
36	4.0	14.0	90.0	108.0	18.0
38	2.0	12.0	70.0	84.0	14.0
40	0.0	10.0	50.0	60.0	10.0
42	0.0	~~6.7~~ 8.0	32.0	40.0	6.7
44	0.0	~~4.0~~ 6.0	18.0	24.0	4.0
46	0.0	~~2.0~~ 4.0	8.0	12.0	2.0
48	0.0	~~0.7~~ 2.0	2.0	4.0	0.7
50	0.0	0.0		0.0	0.0

It is possible to make certain "running adjustments," during the process of routing the triangular hydrograph of Fig. 12, to eliminate the effect of the discrepancies already mentioned. Table 2 presents the calculations by means of which this is accomplished. The general procedure consists of routing by the authors' method except that, for each impossible value of outflow which is obtained, the nearest possible value is substituted. The following nomenclature is used: I = total flow into reach, entering entirely at upper end, 1,000 cu ft per sec; O = outflow in 1,000 cu ft per sec; and S = channel storage in 1,000 sec-ft-hr. In Table 2, the fifth column contains progressive values of

$$S_0 + I_0 + 2\, I_1 - O_0 = S_1 + I_1 + O_1 \dots\dots\dots\dots\dots (11)$$

which corresponds to Eq. 4 of the paper. Assuming that S = function $(I + O)$, the uniform-flow-versus-storage relation which has been assumed for the reach

gives

$$S = 5\,(I + O) \qquad (12)$$

It follows that the relation required for obtaining the sixth column of Table 2 by the authors' method is

$$I + O = \tfrac{1}{6}\,(S + I + O) \qquad (13)$$

When the values of storage in Table 2 are plotted against simultaneous values of $(I + O)$, the result shown in Fig. 13 is obtained. It seems as if the effect of the writer's "adjustments" may have been in the direction of "correcting" the relation between storage and $(I + O)$ for the fact that the variation of flow between inlet and outlet is concave upward from Hour 0 to Hour 10, and from Hour 40 to Hour 50, while being concave downward from Hour 20 to Hour 30. The resulting hydrograph of outflow is shown in Fig. 12 by the short dashes. It is seen to be a perfect image of the inflow hydrograph, with a time lag of 10 hr.

It is generally agreed that if, at all sections of a reach, discharge were entirely dependent on stage, the outflow and inflow hydrographs would be identical in shape. Although this condition is sometimes approached in natural river reaches, it should not be assumed to represent the general case. Since the assumption that storage is a function of average flow in the reach obviously implies such a condition, a method of flood-routing based on that assumption is not adaptable to general application.

Several routing methods used in present practice take care of the individualities of any given reach by means of factors embodied in the procedure. These factors, however, are generally unreliable unless they are evaluated from analysis of at least one flood, for which hydrographs of total inflow and outflow are either known or obtainable by estimation. The writer believes, however, that a practical method of flood routing, the required factors in which can be determined directly from the measurable physical properties of the reach, can be developed.

In the meantime, it is suggested that a more satisfactory solution of the authors' problems could perhaps be effected by the following procedure:

1. Using the procedure outlined in part I of this discussion, obtain the hydrograph of channel storage, for the flood chosen for study.

2. Plotting values of outflow against simultaneous values of storage should give a loop showing a larger volume of storage for a given discharge during rising stages than during falling stages. By plotting values of storage against later values of outflow, a number of trials will indicate what time interval between storage and outflow effects the best closure of the loop into a curve.

3. The inflow hydrograph could then be routed according to a method presented by Otto H. Meyer,[19] using a "storage lag" as nearly as possible equal to the time interval which effected best loop closure. The result should be a

[19] "Simplified Flood Routing," by Otto H. Meyer, *Civil Engineering*, May, 1941, pp. 306–307.

more or less satisfactory outflow hydrograph, depending on the reliability of the data used in previous steps. Incidentally, the process of obtaining the storage-outflow relation and storage lag is described in more detail by Mr. Meyer in the publication cited.[19]

4. Using the same routing procedure, outflow may then be computed for any recorded or assumed total inflow, or by reversing the routing procedure, total inflow may be computed for any recorded or assumed outflow.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CAVITATION IN OUTLET CONDUITS OF HIGH DAMS

Discussion

By J. H. Douma, Jun. Am. Soc. C. E.

J. H. Douma,[32] Jun. Am. Soc. C. E. (by letter).[32a]—Undoubtedly, this paper has been read with much interest by hydraulic engineers who have been concerned with the cavitation problem. It is especially noteworthy for being one of the first in this relatively new field of model investigation. In addition to a detailed outline of the procedure in conducting a model study for an investigation of cavitation, the authors have presented data that are useful in some degree to designing engineers. No attempt, however, has been made to establish a design criterion for outlet conduit entrance curves. Such a criterion would eliminate the seeming necessity for a model study of every proposed design.

For any given high velocity of flow, the boundary conditions of a closed system may be so varied that a pressure intensity equal to the vapor pressure will be approached at some point on the boundary. When the pressure intensity is reduced to the vapor pressure at any point, the liquid will just begin to vaporize. The region of vaporization will grow in size as the pressure intensity is further reduced, and cavities composed of water vapor and flying particles of water will form. The abrupt collapse of these cavities as they are carried into regions of higher pressure intensities causes the destructive disintegration of boundary surfaces known as pitting.

To prevent the formation of cavities the boundary conditions must be such that the outermost streamlines of the resulting flow conform approximately with the boundary profile over their entire length. This requirement can be fulfilled by proportioning any conduit inlet passage to fit the shape of a jet issuing from a sharp-edged orifice of the same cross section as the maximum inlet section. The effect, on the flow, of entrance surfaces so designed is to

Note.—This paper by Harold A. Thomas, M. Am. Soc. C. E., and Emil P. Schuleen, Assoc. M. Am. Soc. C. E., was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Jerome Fee, Assoc. M. Am. Soc. C. E.; April, 1941, by Messrs. V. E. Leman, P. S. O'Shaughnessy and E. S. Randolph, and Carroll F. Merriam; May, 1941, by Hunter Rouse, Assoc. M. Am. Soc. C. E.; and June, 1941, by Messrs. G. H. Hickox, and J. M. Mousson.

[32] Asst. Hydr. Engr., U. S. Engr. Office, Los Angeles, Calif.

[32a] Received by the Secretary June 18, 1941.

modify the outermost streamlines so that positive pressure intensities exist at all points on the boundary surfaces.

In Fig. 19, the upper surfaces of free jets issuing from a sharp-edged circular orifice and from a long narrow slit are shown plotted for a unit *vena contracta* thickness. With the origin of coordinates at the orifice edge, x and y are the horizontal and vertical distances from the origin, and D is the jet thickness at the *vena contracta.*

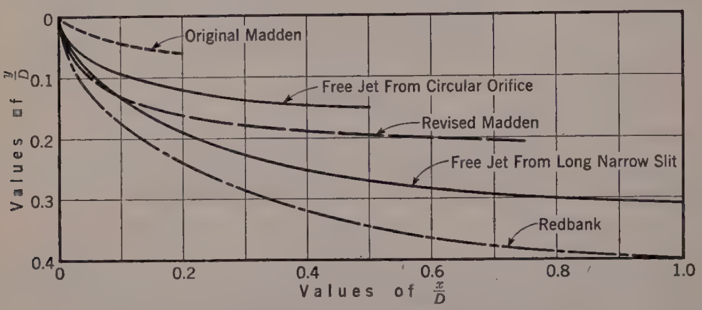

FIG. 19.—COMPARISON OF UNIT ENTRANCE CURVES

For comparative purposes, similar unit top curves of the original and revised entrance sections of the Madden Dam conduit, and of the final design of the entrance section of the Redbank Dam conduit, are also plotted in Fig. 19. The original Madden Dam curve falls far short of the free-jet curves, a condition conducive to the development of cavitation, which is borne out by the severe pitting of the prototype conduits. The revised entrance curve for Madden Dam, shown to have no cavitation potentialities by the model study, approximates the free-jet curve for a slit at its upstream end and lies between the free-jet curves for the circular orifice and the slit at its downstream end. The Redbank curve lies well beyond the free-jet curves, and a model study indicated satisfactory performance with regard to cavitation.

As noted in Fig. 1, the entrances of the Madden Dam conduit are not exactly symmetrical about a horizontal plane through the conduit axis, but the bottom of the stop-log recess terminates in a horizontal step some distance below the beginning of the entrance curve. Lowermost streamlines obviously begin to curve at the upstream end of this shelf. By plotting the circular orifice and slit free-jet curves with their upstream ends at this point, the original Madden Dam bottom entrance curve was found to lie between the free jet curves. No visible cavitation pockets appeared in the stop-log recess during model tests, and, as shown in Fig. 3, no cavitation damage occurred on the bottom of the prototype conduit. Apparently, the recess was filled with liquid, which was effective in easing the streamlines through this region to the entrance curve.

A similar comparison of the side-wall entrance curves with the free-jet curves indicated that they were inadequate to preclude cavitation potentialities, which is substantiated by the cavitation damage occurring at the prototype side-walls. In plotting unit entrance curves for the rectangular conduit, the

conduit height was taken as d for the top and bottom curves, and the width as d for the side curves.

Inasmuch as the jet issuing from a square orifice approaches the shape of the circular orifice jet, and as the opening changes to a narrow rectangular cross section the jet approaches the shape of the slit jet, unit curves for a properly designed entrance to any given rectangular conduit would lie between the circular orifice and slit unit curves. A model investigation of free jets issuing from rectangular sharp-edged orifices, covering a useful range of height to width-of-opening ratio, would provide a basis for determining the exact shape of entrance curves to any rectangular conduit.

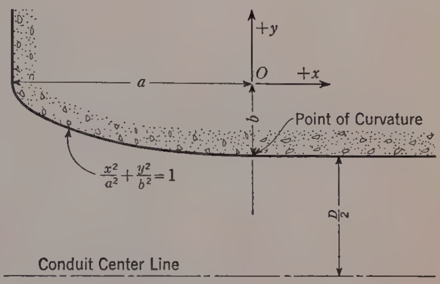

FIG. 20.—ENTRANCE CURVE SHAPE

To evolve a conservative basis of design until definite data relative to rectangular orifice jets are available, the entrance passages to circular conduits might be proportioned to approximate the shape of the free jet issuing from a sharp-edged circular orifice and for rectangular conduits the shape of the jet from a slit. Referring to Fig. 20, the free-jet surface curves approximate an ellipse in shape, expressed in the general form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (30)$$

The *vena contracta* of the circular orifice jet is located 0.5 D from the aperture, and the jet diameter at this point is 0.77 D_o, in which D is the jet diameter at the *vena contracta* and D_o the orifice diameter. This gives a semi-major axis equal to 0.5 D and a semi-minor axis equal to 0.15 D for the surface curve. The design equation of the entrance curve for a circular conduit becomes

$$4\,x^2 + 44.4\,y^2 = D^2 \qquad (31)$$

The location of the *vena contracta* for the slit jet is approximately equal to the *vena contracta* jet thickness from the opening, and the jet thickness at the *vena contracta* is equal to 0.62 times the slit opening. These dimensions give a semi-major axis equal to D, and a semi-minor axis equal to 0.31 D for the surface curve. The design equation for the entrance curves of rectangular conduits becomes

$$x^2 + 10.4\,y^2 = D^2 \qquad (32)$$

in which D is the conduit height for the top and bottom curves and the width for side curves.

The writer contends that Eqs. 31 and 32 can be used reliably, in designing entrances to conduits, with reasonable assurance that cavitation would not occur in the prototype, except in the vicinity of gate slots, if they are not properly designed. As a matter of fact, the adequacy of Eq. 31 already has

been verified by model studies of the entrances to the circular outlets of Grand Coulee Dam. These entrance curves fit closely that given by Eq. 31, and the model study indicated satisfactory performance relative to cavitation by showing positive pressures at all points.

Although, as the authors state, the cavitation phenomenon cannot be studied by means of small-scale undistorted models of the usual laboratory type in which no scale reduction is made in the atmospheric pressure, such models may be used to advantage in checking any conduit design for possible occurrence of cavitation. The method is to locate piezometer openings at numerous points in the region in which cavitation might occur. Then, under the maximum operating conditions, if at least a slight positive pressure is indicated at all points, the design is satisfactory for averting cavitation. This method is ordinarily used by hydraulic laboratories, and, since it results in conservative designs, it is likely that this procedure rather than the so-called vacuum testing method will continue to be favored. A comparison of the required entrance for any given conduit as determined by each of the two methods would be of interest.

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DISCUSSIONS

EVALUATION OF FLOOD LOSSES AND BENEFITS

Discussion

BY MESSRS. H. K. BARROWS, ROGER E. AMIDON, HYMAN J. FINE, AND OTTO F. BUZHARDT

H. K. BARROWS,[10] M. AM. SOC. C. E.[10a]—In his discussion of "Evaluation by Estimating Depreciated Property Values" the author has indicated clearly the objections to using, and difficulties encountered in attempting to use, this method of estimating flood losses and benefits, which he calls the capital-loss method. The writer is quite in accord with his views on this matter and desires to accentuate them.

If the estimates of annual flood losses substantially cover all direct and indirect losses, including proper allowance for any intangible losses, the annual depreciation losses of a property or plant, if correctly determined, will be a reflection of the sum of all these determined losses and hence another determination of the same quantity. In other words, the capital-loss method gives another, less perfect, determination of what has already been obtained by determining average annual losses, based upon the sum of direct, indirect, and intangible losses, and expressed as a capitalized amount for comparison.

In the case of the Connecticut River Flood Control project,[11] in which depreciation was included in estimating flood benefits, the following yearly figures resulted:

Class	Yearly benefits	Ratio to direct benefits
Direct	$ 853,700	1.0
Indirect	808,400	0.94
Depreciation	3,597,000	4.20
Total	$5,259,100	6.15

In this case depreciation was based upon estimates made for the 1936 flood—the largest on record—whereas the direct and indirect benefits were based upon

NOTE.—This paper by Edgar E. Foster, Assoc. M. Am. Soc. C. E., was published in May, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1941, by Messrs. E. L. Chandler, E. F. Chandler, and Charles B. Burdick.

[10] Cons. Engr., Boston, Mass.

[10a] Received by the Secretary August 7, 1941.

[11] H. R. Doc. No. 455, 75th Cong., 2d Session, pp. 108–109.

the average yearly flood over a long period of years. Direct plus indirect yearly benefits total \$1,662,100. Allowance for intangible losses may raise this total to, say, \$2,000,000. The depreciation method, which is (or should be) another determination of the same quantity, gives \$3,597,000. This is obviously not based upon a long-time average flood, but the greatest flood of record, of very recent occurrence, and hence inflated in amount.

The fallacy of adding together two determinations of the same quantity instead of averaging them, if of equal weight in accuracy, is obvious. Considering the uncertainty of the estimated yearly depreciation, it appears that yearly benefits for the Connecticut River project should not rationally be greater than about \$2,000,000. This warrants an expenditure for flood relief of about \$40,000,000, instead of about \$100,000,000, which resulted when depreciation benefits were added in (as was done in the report previously cited, upon which the approval of this flood-control project was based).

The comprehensive flood-relief project for the Connecticut River is now estimated to cost about \$75,000,000. It is clearly of doubtful economy and indicates that the project should not have been approved in the form presented.

The author is to be commended for his excellent presentation of a timely subject.

ROGER E. AMIDON,[12] ASSOC. M. AM. SOC. C. E.[12a]—The evaluation of flood losses and flood-control benefits is an intricate and involved science. The paper by Mr. Foster is comprehensive and as complete a treatise on the subject as the writer has seen. It certainly will be widely used as reference. It approaches the category of a handbook. The author has stated with clarity the enormous number of details involved in an analysis, as well as the indefiniteness of the data upon which every study must be made. Judgment based on experience is rightly given important emphasis.

It is the object of the writer to add a word of caution to the use of the generalized formula in specific cases. The experienced analyst will not be likely to use this information in a manner that was not intended, but the inexperienced analyst may take it too literally and end up with results that are extremely erroneous or, what is more likely, he may become so "bogged down" with inconsistent results that he will throw up his hands in despair.

The watersheds of Southern California are a case in point. The methods of analysis presented by the author could be applied to a study of the Los Angeles River, or any of the larger watersheds with probably fair results. These same methods cannot be applied to the smaller tributaries originating in the steep mountains. Stage-discharge relationships are not applicable to these mountain streams because of the character of the torrential flows of muddy and debris-laden waters that debauch from the canyon mouths on to the alluvial fans below. A comparatively small flow of water may cause damage to areas that have not been damaged by previous major floods. The lodging of a boulder or several logs may cause a stream to leave an apparently well-defined channel and cut through areas that have not been touched by floods since the

[12] Associate Civ. Engr., U. S. Forest Service, Pasadena, Calif

[12a] Received by the Secretary September 2, 1941.

coming of the white man. Denudation by fire may cause a small tributary to go on a rampage from a comparatively small rainfall. With present fire detection and fighting facilities, it is improbable that a sufficient part of a large watershed would be burned at any one time to cause a flood that would be extremely abnormal for the amount of rainfall over the entire area. Even within canyons, above their mouths, the flood flows will meander over bench lands and cause damage to recreation and other developments entirely out of proportion to the stage-discharge relationship. The high percentage of solids in the flows from these canyons causes the fluid to act considerably different from that which normally could be expected from a flow of flood water. Henry B. Lynch,[13] M. Am. Soc. C. E., stated this fact clearly in 1939. In analyzing the economics of an area where such irregular conditions exist, it is necessary to give consideration to these conditions if good results are to be obtained. In areas that are bordered by mountains where denudation by fire is possible and probable, it becomes necessary to correlate the occurrence of fire with the occurrence of flood. It is possible to do this and obtain reasonable results from a study of past occurrences within the area or within adjacent watersheds. In such a comparison it is necessary to make adjustments for differences in values, types of vegetative cover, topography, etc., between the area under analysis and the one with which it is being compared. Values may be adjusted by comparing population or assessed valuation. Areas subject to this type of damage in Southern California are frequently desirable for urban development and consequently may change from an agricultural or waste area to a highly valuable residential section during the period for which the study is being made. Broad experience, judgment, and understanding are essential in making reasonable adjustments to take care of such conditions.

The author's definition of "social benefits" does not describe what the writer prefers to term "Benefits for which it has been impossible to assign monetary values." In special cases these benefits may be so extremely important as to overshadow other benefits. Mountain canyons adjacent to highly developed residential and industrial centers may lose high recreation value as a result of torrential floods. The transformation of such a canyon bottom from a wooded, shady, vegetation-covered glen, to a barren waste of rocks and boulders, is a definite loss to society. Hikers, picnickers, and others seeking outdoor recreation do not gain as much pleasure from the use of an area so ravished. Although the actual number of people using the canyon may not be materially reduced, the pleasure and value to the user are less. If a correct analysis is to be made such flood losses must be given consideration.

The loss of a home and land by people who have given years of their life to their building is usually much greater than any factor commonly applied to the assessor's figures. The writer is familiar with cases in which people have spent several times the market value of their property repairing damage done by a single flood. In one case, about $40,000 was spent on replacing soil, trees, roads, and stream channel on a piece of property that could not be sold on the open market for $15,000. The writer is familiar with many other cases in

[13] "Transient Flood Peaks," by Henry B. Lynch, *Transactions*, Am. Soc. C. E, Vol. 106 (1941), p. 199 (published in *Proceedings* in November, 1939).

which the individual cannot replace damage that was done by floods, but goes on year after year feeling a hurt in his soul each time he looks at the spot where the great oak once stood. This is a value that is not customarily included in flood loss. Sentimental values such as this are essential to human happiness and well being.

Other items exist for which it has been impossible to assign values. Loss of human life as well as loss of wild life is not usually considered in evaluating flood-control measures. Many streams in Southern California have been made useless for fishing as a result of floods.

In the economic analysis of a proposed flood-control program all benefits that will accrue to the proposed measures must be considered and evaluated if the results are to show correct justification. In light of the many indefinite items that combine to make up most studies, such as hydrology, damage relationships, etc., it does not appear to the writer to be such a far step to the monetary evaluation of intangible items such as the aforementioned. Widespread questionnaires, designed and analyzed by competent psychologists, could probably answer, with reasonable accuracy, the question regarding the loss of recreation value. A factor relationship might be developed between measurable and immeasurable benefits by a detailed study by competent technicians in various types of areas. Where measurable values are large, immeasurable values will usually also be large.

It appears to the writer that if flood-control expenditures are to be based on economic justification it will be necessary to set up "yardsticks" of evaluation for all items involved. When this is done in a consistent and uniform manner it will be a step toward the elimination of pressure-group flood-control expenditures. The "cards will be face up on the table" where every one can see the "winning hand." This will be particularly important during the coming years when flood-control projects will be in the foreground for taking up the slack of unemployment resulting from the reduction of the war industry.

HYMAN J. FINE,[14] ASSOC. M. AM. SOC. C. E.[14a]—The importance of the paper by Mr. Foster is readily realized by the fact that the justification for the millions of dollars spent annually on flood-protection projects must be based on a study of the seriousness and extent of the flood problem and on a comparison of the annual charges of the proposed projects with the annual benefits to be derived from them.

As stated in the paper, the average annual flood loss for a particular reach of river is based on: (*a*) The frequency curve for the control gage in the vicinity of the area under study; and (*b*) the stage-damage curve.

Frequency Curves.—On certain streams the available record of flood heights is not sufficiently complete for the preparation of reasonably accurate frequency curves. In many instances, recourse must be made to the flood data available for adjoining watersheds. Furthermore, frequency curves derived by equally acceptable methods of procedure may indicate a variation in the probability of flood stages.

[14] Associate Engr., U. S. Engr., Office, Norfolk, Va.

[14a] Received by the Secretary September 10, 1941.

The modified frequency graph for a certain section of a stream, wherein the floods are reduced by tributary storage reservoirs, depends on the flood routing studies which indicate the effect of such tributary regulation on the main stream flows. Such studies are usually complicated by the lack of adequate hydrological data of past storms for floods of varying magnitude.

The aforementioned factors indicate the importance of extensive hydrological research prior to the preparation of the required frequency curves.

Stage-Damage Curve.—Mr. Foster properly emphasizes the necessity for exercising considerable judgment in the compilation of the data required for the stage-damage curve. In general, reliable information relative to the monetary losses from past floods is not readily obtained unless the floods are of recent occurrence. Furthermore, the establishment of a stage-damage curve, which is accurate throughout the range of probable flood heights, usually requires estimates of the losses for floods higher and lower than those that have occurred recently.

The three classes of property that generally constitute the major part of the damage appraisals to be made in urban areas are large industrial developments, small commercial establishments, and residential property. Large industrial establishments must be covered by separate interviews, and the plant engineer and manager are usually able to furnish the damage data required. In the case of the small commercial establishments or residences, it may be difficult to ascertain the future flood damages to the structures or improvements therein from the owners or representatives inasmuch as these people usually do not have sufficient knowledge of the cost of repairs. Such information can be obtained more readily from other sources.

TABLE 3.—Appraisal of Flood Damages in Zone 1, Roanoke, Va., for the October, 1937, Flood and for Probable Floods Above and Below the 1937 Stage

(Mr. John R. Scott, Owner, Gille Clothing Co., 601 Chestnut Street, Roanoke, Va. [Left Bank of Stream])

Item of damage	Feet Above or Below the 1937 Stage:[a]									
	−8[b]	−6	−4[c]	−2	0	+2	+4	+6[d]	+8	+10
Direct Damage										
Structure and improvements			0	100	300	500	600	700	1,000	1,100
Machinery			0	50	200	350	350	350	600	700
Stock				0	400	600	600	600	1,200	1,500
Miscellaneous										
Total			0	150	900	1,450	1,550	1,650	2,800	3,300
Total indirect damage			0	100	300	440	500	580	1,120	1,320

[a] Elevation of 1937 stage in vicinity of establishment is 862.5 ft above mean sea level. [b] Ground. [c] First floor. [d] Second floor.

As an example, Table 3 illustrates the collection of flood-damage data for a small commercial concern. The damage to the stock has been obtained from the proprietor who is thoroughly capable of furnishing such information. One important factor to take into consideration in estimating the stock damage is

the time of flood warning available to the community, wherein this concern is located, so that the owner may raise his stock to a height higher than the crest of the anticipated flood. Damages to structures and improvements have been obtained from a curve indicating the cost of repairing floors, walls, plaster, fixtures, heating system, refrigerators, windows, doors, etc., for various heights of flooding. This latter curve is based on data furnished by contractors who perform such work. The repair costs should be reduced by a depreciation factor to determine the actual flood loss to the business establishment.

The indirect loss is expressed by the following formula:

$$\text{Indirect loss} = N\,S\,P \qquad (2)$$

in which: N = number of days the establishment must be closed due to evacuation and rehabilitation following the flood; S = average daily sales lost during the flood period; and P = average gross markup per dollar of sale (including overhead, labor, and profit). A curve indicating the number of days of shutdown and rehabilitation for small business concerns can usually be established from actual flood data and from information furnished by contractors relative to time involved for repairs, etc. The average daily sales and the average gross markup can be obtained readily from the various business establishments affected by floods.

Agricultural Losses.—With reference to the method of deriving agricultural losses explained in the paper, it is believed that one additional item, which may be referred to as the inundation factor, should be taken into consideration and listed as item (*h*). This factor is based on the best available damage data for past floods of varying stages and may be expressed as follows: Inundation factor (for a flood of certain magnitude occurring in a particular month) $= \dfrac{\text{actual crop loss per acre}}{\text{maximum possible crop loss per acre for that month}}$. The total crop loss resulting from a flood of certain magnitude can now be expressed by

$$\text{Crop loss} = I\,(A\,C_1\,S_1\,L_1 + A\,C_2\,S_2\,L_2 + A\,C_3\,S_3\,L_3 + \cdots) \qquad (3)$$

in which: A = total cultivated area inundated; C = percentage of cultivated area in a certain crop; S = sales value of that crop per acre; L = maximum possible crop loss in the month that the flood occurred, in percentage of the sales value; and I = inundation factor. The ($S\,L$)-value would be obtained from a graph similar to that in Fig. 4.

If the inundation factor is taken into consideration, the crop loss value of \$12.62 shown in Table 2 would not remain a constant for all stages. Depending on the topography of the area inundated, the crop loss would probably be a minimum at the lower stages, would gradually increase to a maximum at the medium stages, and then would remain at a maximum or possibly reduce slightly at the higher stages.

Annual Benefits Attributable to Local Protection.—Local protection improvements will usually eliminate all the losses caused by inundation up to the stage at which the flood-prevention structures are overtopped. Floods higher than those against which protection have been provided have a certain probability of occurrence within the life of the protection works and may destroy or impair

the usefulness of these structures. Therefore, it is logical to deduct, from the annual benefits, the incremental annual damages caused by such floods over what they would normally have caused without the construction of the flood prevention works.

Standardization of Data.—It is believed desirable to standardize on certain of the data required for a flood study. Stage-damage curves above first-floor levels for certain types of property, monthly crop-loss graphs, and other similar data, if available in standard form, would obviate the necessity of extensive studies for the preparation of such graphs.

Mr. Foster's paper is a distinct contribution toward the determination of the average annual losses resulting from floods and the benefits to be derived from flood-prevention structures.

OTTO F. BUZHARDT,[15] ESQ.[15a]—A clear picture of the economics of flood control is presented in this paper. One point of special interest is the emphasis placed on indirect and unique losses and damages. It is usually a relatively simple matter to estimate physical flood losses or damages, but to evaluate indirect losses properly is often more difficult. These losses often exceed direct damages.

As Mr. Foster stated, stage-area and stage-damage curves may be derived through the use of information obtained through flood damage surveys. As a matter of fact, all estimations of flood damages and flood-control benefits depend primarily on field surveys of the flooded areas. When used in conjunction with the record of a conveniently located gage, areas and damages

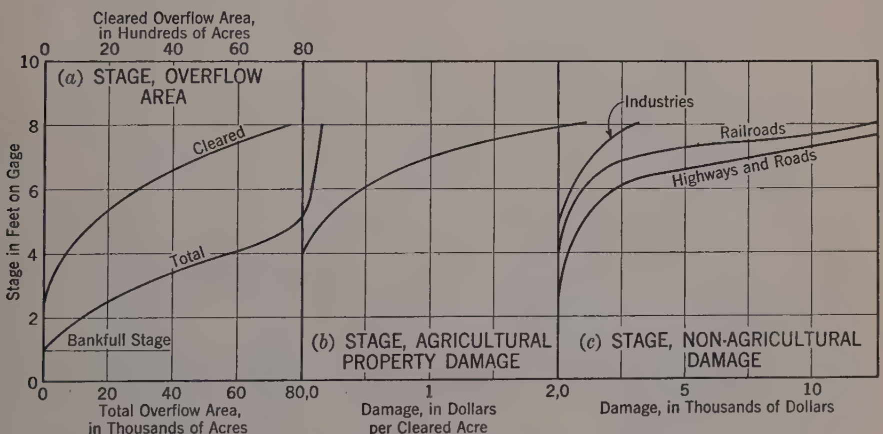


FIG. 6.—DAMAGE AT VARIOUS FLOOD STAGES

other than to crops may be determined readily for any stage. Typical examples of these curves are given in the following:

Fig. 6(*a*) shows the areas flooded, subdivided into cleared and total areas for respective gage heights. It will be noted that the cleared area usually

[15] U. S. Engr. Office, Vicksburg, Miss.

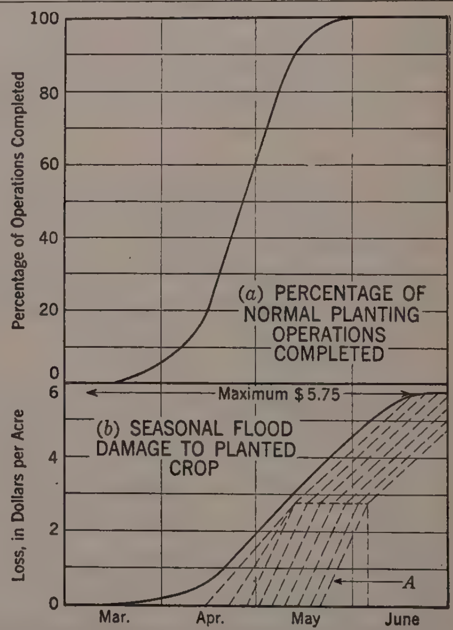
[15a] Received by the Secretary September 15, 1941.

begins well above bankfull stage because of the high frequency of overflow at the lower elevations. Naturally, under varying conditions, the areas that were inundated for a certain gage height may not be precisely the same but in general the difference will compensate.

The author asserts correctly that agricultural losses, other than crops, may be treated as urban losses, but it is logical to place damages of this kind on a per-cleared-acre basis (see Fig. 6(*b*)) since the types and extent of improvements primarily depend upon the cleared farm area. Usually these curves have a tendency to flatten out for higher stages due to the fact that improvements are located on higher parts of the flood plains. Mr. Foster has a good point in

TABLE 4.—Derivation of Factors Used in Plotting Curve for Damages to Prepared Ground and Planted Crop

First date of flood	Preparing and Planting		Other Operations		Total damages
	% completed	Loss per acre	% completed	Loss per acre	
March 15	0	0.00	0	0.00	0.00
April 1	6	0.17	0	0.00	0.17
April 15	20	0.55	0	0.00	0.55
May 1	63	1.73	6	0.18	1.91
May 15	94	2.58	20	0.60	3.18
June 1	100	2.75	63	1.80	4.55
June 15	100	2.75	94	2.82	5.57
July 1	100	2.75	100	3.00	5.75



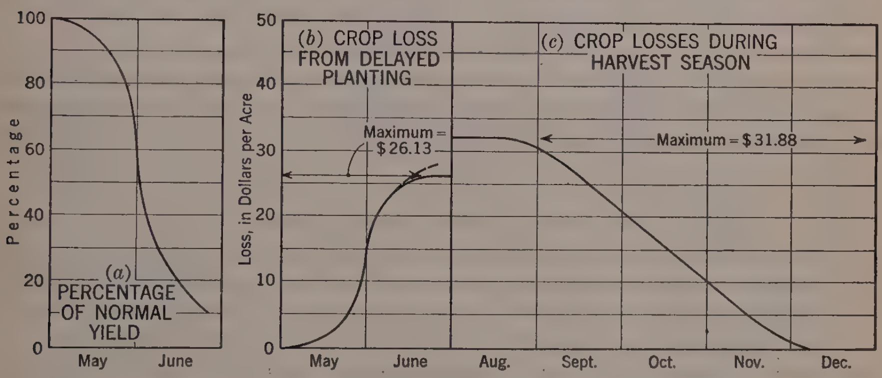
that, as a matter of conservatism, the maximum estimated damage for any property-value curve should be limited to that for which an actual computation has been made.

Stage-Nonagricultural Property Damages.—In order to construct a curve such as the stage-damage curve for Lawrence, Mass., damages such as those of Fig. 6(*c*) must first be ascertained. It is important to note Mr. Foster's explanation that the composite curve is of direct damages. Since indirect damages depend on factors other than stage, their values may not be a part of this or similar curves.

The author has summed up very neatly the data needed for the construction of crop curves, but since agricultural damages in some parts of the United

States constitute the greater part of losses, it is believed that a more detailed explanation of the derivation of these curves is needed. Accurate estimates of crop losses are difficult to compute because of varying types of farming and crop distribution. Topography, as well as the sectional locality, determines the type of equipment used on farms, with a resulting difference in production costs. For this reason the basic information used in the following curves must apply to a specific area, but it is believed that the principle may be applied to a crop loss in any locality. Although the areas encountered support a variety of crops with a wide range in gross value of yields, curves relating to cotton crop damages are used for illustrative purposes.

TABLE 5.—Derivation of Factors Used in Plotting Curve for Crop Losses



% yield[a]	Cotton yield[b] (lb)	Production Costs, in Dollars: Preparing[c]	Over head[d]	Total, Cols. 3 and 4	Harvesting and ginning	Total	Value of yield	Col. 8 minus Col. 7	Profit[e]	Date
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
100	375	5.75	20.25	26.00	15.00	41.00	46.88	5.88	0	April 20
90	340	5.75	20.25	26.00	13.50	39.50	42.20	2.70	− 3.18	May 20
80	300	5.75	20.25	26.00	12.00	38.00	37.50	− 0.50	− 6.38	May 26
70	260	5.75	20.25	26.00	10.50	36.50	32.80	− 3.70	− 9.58	May 30
60	225	5.75	20.25	26.00	9.00	35.00	28.10	− 6.90	−12.78	May 31
50	190	5.75	20.25	26.00	7.50	33.50	23.40	−10.10	−15.98	June 1
40	150	5.75	20.25	26.00	6.00	32.00	18.75	−13.25	−19.13	June 3
30	110	5.75	20.25	26.00	4.50	30.50	14.05	−16.45	−22.33	June 8
20	75	5.75	20.25	26.00	3.00	29.00	9.40	−19.60	−25.48	June 15
10	40	5.75	20.25	26.00	1.50	27.50	4.70	−22.80	−28.68	June 25
0	0	5.75	20.25	26.00	0	26.00	0	−26.00	−31.88	....

[a] From curve (*a*). [b] Lint. [c] Includes planting, cultivating, and poisoning. [d] Includes work stock care. [e] Col. 9 minus 100% potential loss ($5.88), to be used in plotting curve (*b*).

Curve (*a*) in Table 4 shows the average first date (March 15) that cotton will be planted, with normal planting variations between these dates. This curve is considered with normal conditions only and is an integral part of curve (*b*), Table 4.

As Mr. Foster states, it is assumed that crops are destroyed when lands are flooded, regardless of the depth of water; therefore, the season of flooding is the governing factor in estimating crop losses. Curve (*b*), Table 4, the planted crop damage curve, is designed to show the accumulated expense incurred in planting and cultivating one acre of cotton. These values are referred to days of the year in order that planted crop damages may be obtained for any day on which an overflow may occur. Should a flood destroy a crop between March 15 and July 1, the resulting damage would be the amount corresponding to the date of flooding. If this flood occurred at such a date that reasonable return could not be expected from a replanted crop, then floods subsequent to that date would naturally not damage crops below the elevation of the previous flood. In general, season permitting, crops are replanted within approximately ten days after lands are free from flood waters, the cost being almost equal to that of the first planting. As the season progresses, the speed of replanting operations is increased as shown by the broken lines "*A*" of curve (*b*)(Table 4). In estimations for a series of floods, the damages to a replanted crop are indicated by lines "*A*," the initial point falling ten days after the preceding flood. Consideration has been taken of the fact that flood water will gradually recede to bankfull stage, and it is assumed that a farmer will plant from the higher to lower elevations.

Curve (*a*), Table 5, shows the decline of production with respect to dates of late crop planting. This curve is used in deriving factors for curve (*b*), Table 5, "seasonal value of crop loss from delayed planting."

Crops destroyed by floods and planted after the average date of April 30 are usually reduced in production value. It is logical that a farmer will not continue replanting to a date beyond which his return is less than his cost of overhead and work stock care plus his normal net gain. Curve (*b*), Table 5, shows the rate of decline in production value. Curve (*c*), Table 5, "crop losses during harvest season," shows values of the crop from the time the crop is "laid by" through the harvest season. The maximum value shown in the curve is the sum of maximum values found in curves (*b*), Tables 4 and 5. When these curves, ((*b*), Table 4; and (*b*) and (*c*), Table 5) are combined, a curve such as Fig. 4 of the paper will be the result.

Under the heading "Benefits from Flood Control" it should be stated that channel improvement is used for stage reduction, as are reservoirs. The stages are reduced through better drainage facilities and as a rule the duration of flooding will be shorter. The entire flood will be moved forward—the first day of flooding occurring possibly a day or more earlier than it would have in the unimproved channel and the last day of flooding will probably be several days earlier, depending, of course, on the individual flood.

Contrary to Mr. Foster's view, the writer believes that the total benefits derived from a flood-control project should be: (1) Flood damages prevented, through protection, plus an increase to allow for future development of the area considered, without protection; (2) enhancement to lands already cleared; and (3) enhancement to woodland to be cleared. Some areas are subject to frequent flooding and, as a result, rich alluvial deposits have been built up. As a matter of fact, some farmers find this land so fertile that crops are planted

in spots that are not above a three-year flood frequency. These are unusual cases, of course, and should the land be placed on the market for sale it would be found to have little value. Land values in these areas increase, however, as the elevation increases and the flood frequency decreases. It has been found through field investigation of protected areas, similar to those mentioned herein, that land values will be increased by improved flood conditions. This is due to the probability of increase in the productivity of the land through the use of more economical or intensive cultivation methods and rotation of crops. In estimating the increased value of cleared land on the basis of increased productivity, the lands, under existing conditions, are analyzed to determine the average annual acre cost and income within the area considered. The comparison of these estimates with corresponding values representative of adjacent areas, not subject to frequent overflows, indicates the annual cleared land value to be expected by reason of flood control. If possible, it is preferable to compare the area under consideration with lands already protected by works similar to those proposed, rather than with lands above flood plains where values are dependent upon conditions other than flooding. Since the value of agricultural land varies, in general, according to its productivity or net income, comparison of values before and after protection is indicative of the annual increased land values. This value is exclusive of flood damages prevented.

Annual increased value of woodland, afforded protection, depends primarily upon its value as potential cultivatable land. Flood damages in wooded areas are usually negligible, and the wooded areas receiving benefits through flood control are restricted largely to those areas which might be suitable or desired for crop lands. Generally, parts of wooded areas to be protected are unsuitable for clearing without considerable expenditures for drainage improvement. Certain areas would be retained as timber or woodland for purposes other than crop production. Therefore, the amount of ultimate clearing to be expected in an area, as a result of flood control, may be estimated through comparison with other similar areas having essentially the same degree of protection as proposed for the area considered.

Remembering Mr. Foster's warning that values vary with elevation, for illustrative purposes it will be assumed that lands, in an area under discussion, average \$25 per acre in value. Also, it will be assumed that the average gross value of crops on these cleared lands is equal to approximately 50% of the capital value of land at \$12.50 per acre. Computed on a share rental basis, the net rental income will be approximately \$2.50 or 10% of the capital value of land, from which interest on investment has not been deducted. To the annual gross value of crops (\$12.50 per acre) may be added the annual value of damage to agricultural land that would be prevented by flood protection (\$1.50 per acre for this particular area) the sum of which is the average annual gross value of crops given flood protection, or \$14 per acre. Since the gross value of crops is equivalent to approximately 50% of the capital value of the land, a crop valued at \$14 would be the yield of \$28 land, the \$3 per acre difference between \$25 and \$28 representing the capitalized value of flood damage prevented. However, it has been found that lands of the same type, and with

the same degree of flood protection, have a value of \$35 per acre. From this it is indicated that an average increase in capital value of \$7 per acre would be obtained by flood protection beyond the value of flood-damage elimination, and it may be attributed to more favorable conditions. Based on the 10% expected on this cultivated land, an annual average increase in net return of \$0.70 per acre on increased capital value of land, aside from that due to flood-damage elimination, would result.

In enhancement to lands to be cleared, it is assumed that the present value of unprotected woodland is \$8 per acre, and that the average cost of clearing this land for farming purposes is approximately \$15 per acre. From this, the capital value of the land when cleared must be in excess of \$23 to give a profitable return. Should investigations prove that the ultimate value of land, when cleared, would be \$33 per acre, the indication would be an increased capital value of \$10 per acre if afforded protection. Based on an annual net return of 10%, the average annual net return in this case would be \$1 per acre for the woodland to be cleared.

It should be remembered that the lands described herein are subject to frequent overflows, and values are not based on hysteria caused by an unusual flood, as was suggested by Mr. Foster in his description of the capital value method of land enhancement.

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DISCUSSIONS

METHOD OF PREDICTING THE RUNOFF FROM RAINFALL

Discussion

By Bertram S. Barnes, Assoc. M. Am. Soc. C. E.

Bertram S. Barnes,[5] Assoc. M. Am. Soc. C. E.[5a]—The general applicability of a forecasting procedure developed on one type of stream can be established only by actual trial on other streams. The length of time required for a proper test of the method presented by Messrs. Linsley and Ackermann makes it scarcely practicable for a discusser to present any conclusive experimental results at this time. It is the writer's belief, however, that the scheme calls for a more precise separation of the elements of flow, in which the personal judgment of the computer is not so important a factor.

As a check on the method, the writer has separated the elements of flow in the hydrograph presented by the authors and the results are shown in Fig. 9. Values of the total discharge were picked from the authors' hydrograph (Fig. 2) as accurately as possible and replotted, using a logarithmic discharge scale.[6] All recessions of the form $Q = Q_0 k^t$ therefore appear as straight lines. On account of the seriously overlapping runoff during this short period, and because no other records from this stream were at hand, a certain amount of trial and error was required in establishing the values of the depletion factor k for the three elements of flow. Parts of the graphs that are not marked with circles are either assumed or calculated by the use of k. Points indicated by circles represent values obtained as remainders, by subtraction, and are assumed to contain the errors that occurred in picking the discharge values from the authors' published hydrograph.

The separation shown in Fig. 9 indicates that the ground-water flow was not affected by recharge from the rain event of April 1 and 2 until midnight of April 7, 6½ days after the rain commenced. The recharge from the rain event of April 9 become evident on April 14. For some reason unknown to the writer the effect of recharge from the rain event of April 5 and 6 has become

Note.—This paper by Ray K. Linsley, Jr., and William C. Ackermann, Juniors, Am. Soc. C. E., was published in June, 1941, *Proceedings*.

[5] Hydrologic Supervisor, Upper Mississippi District, U. S. Weather Bureau, Iowa City, Iowa.

[5a] Received by the Secretary August 18, 1941.

[6] "The Structure of Discharge Recession Curves," by B. S. Barnes, *Transactions*, Am. Geophysical Union, 1939, Pt. IV.

merged with that of April 1 and 2, and that part of the ground-water hydrograph (April 7 to 10) does not permit an accurate separation of the amounts of recharge resulting from the individual rains. It appears that the effect of recharge from the second event commenced between midnight of April 8 and

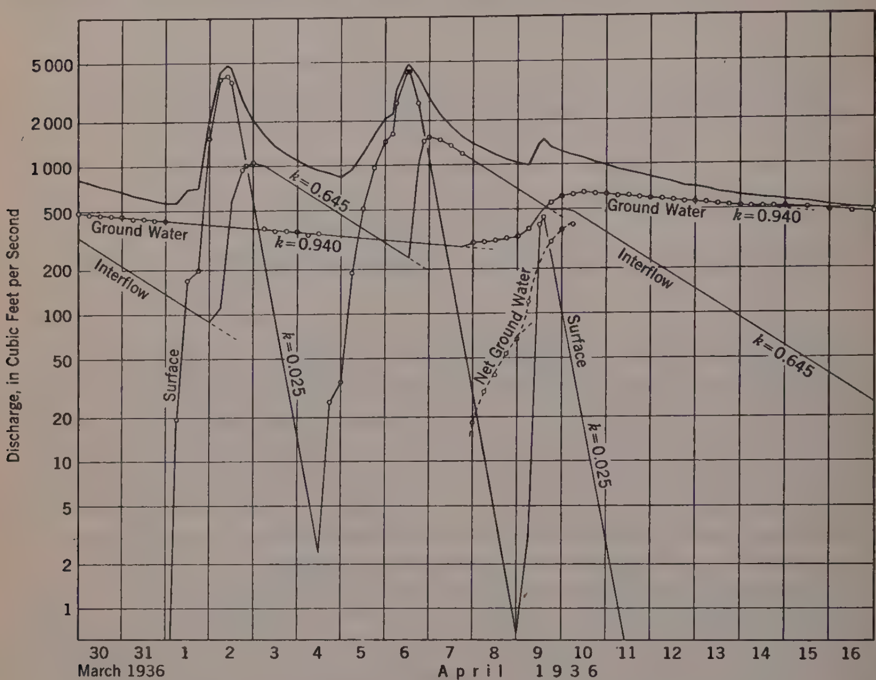

FIG. 9.—SEPARATION OF ELEMENTS OF FLOW, VALLEY RIVER AT TOMOTLA, N. C.

6 a.m. of April 9. It is to be expected that the higher ground-water level resulting from the first recharge would shorten the time lag of runoff from the second. However, the time lag for the beginning of runoff from the rain of April 9 was about 5½ days, with a higher initial ground-water level. As will be shown, the apparent total recharge was greater than could have been produced by either one of the two rain events alone unless the actual rainfall was materially greater than the value that the authors have obtained by Thiessen's method. The graph of net values of the ground-water event of April 7 to 10 is shown in Fig. 9 as a broken line. The curve of the first part of this graph can be extended to separate the two runoff events. If drawn with the maximum plausible curvature, it would give to the first event about one fourth of the total ground-water runoff for both events, and with a minimum curvature, the first event would receive about half the total, or 1.2 in. When comparing rainfall, runoff, and losses, it was found that, if the first event amounted to less than 0.92 in. or 38% of the total ground-water runoff for the two events, the second rain event would show no loss at all. With the assumed maximum

value of 1.2 in. for the first event, the loss from the second rain would be about ½ in. or one third of the total for the two events. It is evident, therefore, that the greater loss occurred from the first rain.

The names used by various writers to designate flows that are neither of ground water nor of surface origin have not been received with general approval. There is also some difference of opinion regarding the cause and nature of such flows. The writer now proposes the name "intermediate flow" or "interflow" and will use the latter term in the present discussion. It was the expressed intention of the authors to combine the elements of interflow and surface flow in the separation that they presented. A comparison of Fig. 2 with Fig. 9 indicates that they have actually included a large part of the interflow in their ground-water hydrograph. It also appears from Fig. 9 and the subsequent tabulation of data that the runoff from interflow is greater than that from surface flow.

It is the writer's opinion that interflows of appreciable amount should be given separate treatment in the forecasting of runoff, for two reasons. In the first place, the volume of interflow appears to be more closely related to the duration of rainfall than is the volume of surface flow or that of ground-water flow. Secondly, the inclusion of variable amounts of interflow with the other elements of flow interferes with the precise analysis of the hydrograph by the unit graph method, by the use of standard depletion curves, or by using the depletion factor.

In this discussion it is assumed that any simple recession of the hydrograph of one element of flow is a curve of the form

$$Q = Q_0 k^t \qquad (1)$$

in which Q_0 and Q are values of the discharge at two instants separated by a time interval t, and k is the depletion factor for the element of flow. The depletion factor is defined as the ratio of a given discharge to that which occurred one time unit previously.

It has been found that Eq. 1 is applicable to all natural stream hydrographs that the writer has attempted to analyze by this means (the largest drainage area being about 4,500 sq miles), except where the hydrograph has been complicated by freezing temperatures or protracted snow melt. J. R. Fleming[7] analyzed a number of periods of record on three streams having drainage areas of 25 sq miles, 320 sq miles, and 2,300 sq miles, respectively, and found that Eq. 1 could be used effectively to separate the elements of flow of all three.

It is true, of course, that evaporation and transpiration from water and land surfaces will affect the hydrograph noticeably at times of low flow. One great advantage of using the depletion factor is that the effect of these losses will be largely eliminated from the calculated volume of runoff. This will simplify the study of the laws governing initial loss, percolation, and surface runoff. The conventional type of depletion curve is affected by all losses that occur during the periods of record used in constructing the curve.

[7] "Analysis of Discharge-Recession Curves for Three Iowa Streams," by J. R. Fleming, unpublished thesis, State Univ. of Iowa, 1941.

The volume of runoff, R, measured on a recession of an element of flow, is expressed as

$$R = \int Q\, dt \quad \text{(2)}$$

Substituting the value of Q from Eq. 1, integrating, and applying the limits zero and T,

$$R = \int_0^T Q_0\, k^t\, dt = \frac{Q_0\,(k^T - 1)}{\log_e k} \quad \text{(3)}$$

When Q_0 is the initial discharge and T is the duration of the period, Eq. 3 is the general expression of the volume of runoff during any period of the recession. Let an infinite time be substituted for T, and Q for Q_0, in Eq. 3. Since k is always smaller than unity, the equation becomes

$$R = \frac{-Q}{\log_e k} \quad \text{(4)}$$

Eq. 4 is the expression of the quantity of water (of one element—surface, interflow, or ground water) remaining in storage at any instant during the recession. It is solved by assigning to k its proper value for the element of flow that is being considered, and evaluating Q.

The total volume of water that a given rain event contributes to one element of flow may be calculated from the hydrograph as follows:

(1) Determine the hydrograph of the element of flow by graphical analysis, tabular computation, or a combination of the two. This graph should be drawn to a point that is past the peak and far enough down the recession limb so that Eq. 1 is applicable. Subtract the amounts of flow that are the result of previous rain events. These may be calculated by the use of logarithms, or the previous recession may be prolonged as a straight line on semilogarithmic paper and the values picked off directly. The depletion factor k is evaluated by taking values of Q at any two points on the recession.

$$\text{Log } k = \frac{(\log Q_1 - \log Q_2)}{T} \quad \text{(5)}$$

in which T is the elapsed time between the two points selected. In this discussion, one day will be used as the unit of time.

(2) Select a point on the hydrograph that is well past the peak of the event and definitely on the recession limb. Calculate the net runoff up to that point in the usual manner by the summation of values of $Q\,t$ for periods of one hour, one day, or any other convenient basis. In Eq. 4, substitute for Q the ordinate of the discharge hydrograph at the point selected and determine the value of R. Add the result to the volume of runoff previously computed for the initial part of the event. The sum will be the total volume of water that the storm has contributed to that element of flow.

The results of the foregoing analysis, compared with that of the paper, are presented in Table 2 with those obtained by the authors. The values of the

depletion factor k were found to be as follows: Surface flow, $k = 0.025$; interflow, $k = 0.645$; and ground-water flow, $k = 0.940$. It is probable that a more accurate determination of these values could have been made if a longer period of record than 18 days had been used. The consistent plotting of the remainders in Fig. 9 indicates that the adopted values are sufficiently accurate for the present purpose.

TABLE 2.—TABULATION OF DATA AND COMPUTATION OF LOSS

(Quantities of Water are Expressed in Inches)

Date (April, 1936)	Rainfall	Duration (hours)	Runoff, surface (writer)	Runoff, interflow (writer)	Runoff, Ground Water		Runoff, Surface Plus Interflow		Runoff, Total		Loss	
					Author	Writer	Author	Writer	Author	Writer	Author	Writer
1............	4.30	20	0.98	0.96	1.24	(1.1)	1.71	1.94	2.95	(3.04)	1.35	(1.26)
5............	3.88	32	1.17	1.23	1.38	(1.3)	1.42	2.40	2.80	(3.70)	1.08	(0.18)
Total, 1 and 5....	8.18	52	2.15	2.19	2.62	2.40	3.13	4.34	5.75	6.74	2.43	1.44
9............	1.47	10	0.07	0.08	0.70	0.19	0.09	0.15	0.79	0.34	0.68	1.13
Total, 1, 5, and 9.	9.65	62	2.22	2.27	3.32	2.59	3.22	4.49	6.54	7.08	3.11	2.57

Quantities enclosed in parentheses depend upon an assumed amount of 1.1 in. for the ground-water runoff resulting from the rain of April 8 and 9. This value has previously been shown to lie between 0.92 in. and 1.2 in. The total amounts of runoff and loss shown in Table 2 are those resulting only from the three rain events listed. Two other small rains occurred during the period but are assumed to have produced no runoff or recharge to ground water. There was also runoff resulting from precipitation that occurred previous to March 30. At midnight of March 29 there was an interflow of 330 cu ft per sec, corresponding to an amount of 0.25 in. in storage. It seems likely that this flow resulted from the thawing of frost in the soil or of snow in gullies and depressions. There was a ground-water flow of 480 cu ft per sec, corresponding to an amount of 2.61 in. in storage.

The storage equation may be applied to these data, using the form:

$$\text{Rainfall} - \text{runoff} - \text{loss} = \text{change in contents} \quad \ldots\ldots\ldots\ldots (6)$$

The total loss shown in Table 2, therefore, may be checked with reasonable accuracy by the following summary, using storage values determined by Eq. 4:

Description	Loss (inches)
Total rainfall, March 30 to April 16, inclusive........	9.70
Total runoff, March 30 to April 16, inclusive.........	7.42
Amount in storage at midnight, March 29...........	2.86
Amount in storage at midnight, April 16............	2.54
Decrease in storage, March 30 to April 16, inclusive..	0.32
Total loss, March 30 to April 16, inclusive...........	2.60

The writer believes that the suggestions contained in this discussion will not only simplify the work required in forecasting by methods similar to that presented by the authors, but will also eliminate, to a large extent, the errors

arising from personal judgment in separating the elements of flow. The precise separation of the three elements should make each of them more readily predictable in amount. Whenever two elements of flow are added together in unknown proportions, the forecasting of their total amount and rate of depletion is necessarily complicated and uncertain, especially since a set of natural conditions favorable to a large flow of one element will not necessarily favor a large flow of either of the others.

Specifically, the application of the principle of the depletion factor would make straight lines of the curves given in Fig. 3. The slope of each line would be equal to the reciprocal of the appropriate depletion factor. It has been the writer's experience that ground-water curves of the type shown in Fig. 3(*a*), when extended downward to zero discharge, do not ordinarily pass through the origin. This fact suggests that if today's ground-water flow were zero, tomorrow's would be negative; that is, if any water were to be introduced into the stream channel tomorrow, it would be subject to a loss into the stream bed.

Likewise, Fig. 4 would become a single straight line with values of the net increment of ground-water discharge for its ordinates and recharge to ground-water storage for abscissas. Similar diagrams could be drawn for surface flow and interflow, but such diagrams would have to be used with care and skill in order that the proper allowance might be made for runoff occurring previous to the peak.

The authors' procedure of separating initial loss into two parts—surface loss and field-moisture deficiency—and comparing the latter with losses from an evaporation pan, is a distinct departure from any previous practice known to the writer. It is believed that the plan merits trial under various conditions of climate, soil, agricultural development, and topography. The degree of development of growing plants, especially annual crops, may have to be considered when comparing evaporation and transpiration from a natural watershed with the measured evaporation from a standard pan.